SOME WELFARE IMPLICATIONS OF THE RIGHT TO
BUY BACK PREMATURE DEBT

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Abstract:
This thesis seeks to answer the question whether or not debtors should be given the right to buy back debt
prematurely at the opportunity cost of the creditor.

Authorship declaration:
I hereby declare and confirm that this thesis is entirely the result of my own work except where otherwise
indicated. I acknowledge the supervision and guidance I have received from Prof. dr. W. Weigel. This thesis is
not used as part of any other examination and has not yet been published.

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Introduction

“The question is commonly thought of as one in which A inflicts harm on B and what has to be decided is: how should we restrain A? But this is wrong. We are dealing with a problem of reciprocal nature. To avoid the harm to B would inflict harm on A. The real question that has to be decided is: should A be allowed to harm B or should B be allowed to harm A? The problem is to avoid the more serious harm.”

Coase (1960)

Of debt and mirrors

Whenever a creditor makes a loan to a debtor, the creditor acquires a right to be repaid; similarly, whenever a debtor issues a bond to a creditor, the debtor gives up a right and is obliged to repay. If the two parties are otherwise equal, then the debtor’s balance sheet can be said to be a mirror-image of the creditor’s. And just like with a mirror, it is not left and right that are switched, but rather, the position of each with respect to a contract: it is this that gives the appearance of a shift between debit and credit. For, if the value of a bond is € 100, then the creditor’s position is + € 100, and the debtor’s position is — € 100. Or, when you stand in front of the mirror, with in your outstretched right hand a € 100 note, you simultaneously see your mirror-image holding this € 100 note with its left hand: if you could stretch out your hand through the mirror, then you could hand over the note.

This symmetry\(^1\) however breaks down, when the creditor, is allowed to sell this right to another; for what could conceivably be the action of the debtor that mirrors the action of the creditor? Imagine yourself again in front of that mirror, but now instead of handing that object to your mirror-image, you hand it to someone at your side of the mirror.

\(^1\) For a very clear definition of symmetry see Feynman (1964) 52-1: “A thing is symmetrical if there is something we can do to it so that after we have done it, it looks the same as it did before".
This thesis is premised on the idea that symmetry, for one reason or another, is important; the main question of this thesis evolved out of the curiosity what you as the debtor would be handing over and to whom. The debtor in this question would be selling the obligation to repay, meaning that the debtor would promise to repay to someone else. For the symmetry to hold, the right of the creditor to sell the right to repay has to be mirrored by the debtor’s right to sell the obligation to do so. The solution to this problem is simple, as the creditor has the right to demand repayment, the debtor should have the right to fulfill the obligation. Now, before you sell the obligation, turn 90 degrees counterclockwise: what if by selling the other obligation, you fulfill the old one? The creditor, instead of selling the right to repay, now accepts the bond in fulfillment of the old one.

This story started out by noting that a debtor and a creditor differ only in their position with respect to a contract; the mirror in many ways is a metaphor for what McCloskey calls “the so-called Coase-theorem.” In a perfectly competitive market, price is equal to the best foregone alternative, everyone knows everything, and transaction costs are zero: the allocation of rights is irrelevant, and your mirror-image is otherwise identical. What happened above is that the debtor bought back the debt at the opportunity cost to the creditor.

The main question this work is concerned with is, whether this symmetry is desirable in reality, and whether thus debtors should or should not be able to re-purchase debt at the opportunity cost to the creditor. In what other ways do debtors differ from creditors? This is what the Coase Theorem is about: why does symmetry break down and how should we deal with it? \(^3\)

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1. McCloskey (1998) argues that economists have gotten Coase (1960) exactly backwards. What erroneously has been called the Coase theorem is neither a theorem nor what Coase (1960) is about. McCloskey (1998) p.368: “Coase’s actual point, the core of a Coasean economics, was to note what happens in the many important cases in which transaction costs cannot be neglected.” In other words the theorem is the same; the difference is in how you look at the world.

2. There is an analogy to be made with physics in a similar vein as Calabresi & Melamed (1972) p.1095 do at the start of their paper where they likened the perfectly competitive model of economic theory to a world without friction in physics. In physics, the symmetry of fundamental laws refers to observation that the same laws seem to apply at different points in space and time. Anderson (1972) p.394 gives the
Research question

Each thesis needs a question to investigate and provide an answer to so as to structure the topic we deal with. The question that this thesis seeks to answer is the following:

“What, if any, are the economic justifications for (not) giving debtors the right to buy back pre-mature debt at the opportunity cost to the creditor?”

The reason for asking this question is the one above; when should this right be there, when should it not, and why? A secondary reason is that there are obvious parallels from this particular debt contract to contracts in general; for the right to re-purchase is equivalent to the right to breach and substitute damages instead. Therefore, the same or similar answers should apply to whether and when breach should be allowed.

Structure

In order to answer the question two preliminary questions will have to be dealt with. First, what is it that we’re dealing with, or what is the economic concept of debt? This is the subject for the first chapter. This chapter is a long one for it introduces the concepts, the framework, but also the assumptions that will be used throughout the thesis. The second question is what the terms example of Newton “who may have asked himself the question: What if the matter here in my hand obeys the same laws as that up in the sky—that is, what if space and matter are homogeneous and isotropic?” and discovered gravity. When symmetry breaks down, however, this prompts the question of why that is so. Feynman (1964) p.52-12 gives the example of why the orbit of a planet is not a circle but rather an ellipse. In economics the analogy is not with economic laws and physical laws and such – symmetry in one aspect does not imply that the disciplines are symmetrical in all aspects down to their very concepts – rather there is an analogy when it comes to the breakdown in symmetry as an engine of inquiry. So instead of stating that there must be a friction when the allocation of rights matters, we could also say that there must be an asymmetry (see also Akerlof 1970 p.498; “An asymmetry in available information has developed”).

4 An example of a lack of symmetry as a source of inspiration can be found in Calabresi & Melamed (1972) pp. 1115-1116.
are of the optimal debt contract, or in short: what does the optimal debt contract look like? The answer to this question will provide the main part of the model and a benchmark from which to evaluate the existence of the right to re-purchase. For, the right to re-purchase can affect what the terms of the optimal debt contract.

After the preliminary questions have been dealt with, the third chapter will provide an answer to the question and will conclude the thesis with some simple lessons.
Chapter 1: Introduction to debt

And forgive us our debts, as we forgive our debtors

Matthew 6:12

1.1 What is debt?

This is a thesis about debt and the buy-back of premature debt in particular. It is a good custom to start such a work by delineating the subject in the introduction of that work by means of a definition or: what is debt?

One way of answering this question is to look up the etymology or ‘origin’ of the word. This exercise would tell us that the word debt comes from the Latin debitum derived from the verb debere meaning ‘to owe’ and referring to an obligation that can be monetary, in kind, moral or some combination of the three. In the New Testament (see quote above) debt has all three meanings; the original Greek for debts, τα όφειλήματα, is derived from the verb ὀφείλω with a similar meaning and originally reserved for only the legal and economic sphere (Sedláček 2011 p.134/p.167). Another way of delineating the subject, closely related to the above, is to examine the history of the phenomenon in the hope that the debt-relations of the past will tell you something about those in the present and those of the future. For example (Graeber 2011 p.212):

“There is a shape to the past, and it is only by understanding it that we can begin to have a sense of the historical opportunities that exist in the present”

Let $D^t$ denote the set containing the collection of elements that characterize a debt-relation at time $t$, then this second approach amounts to the claim that $D^t \approx D^{t+s}$ for all $s$. The assumption is now that what characterized a debt-relation in the past will characterize it today: the elements
contained in each set are (roughly) the same. This assumption is also known as the *genetic fallacy.*
The first approach can now also be expressed; let \( d^j_t \) denote a subset \( j \) of \( D^t \) at time \( t \), then each \( j \) can be set to correspond to the different elements used to construct a particular meaning of the concept *debt* for any \( t \). The problem with the first approach now also becomes apparent: which subset should you choose and is the correct choice even mentioned above?

Contrast this with a functional approach, and in particular the economic approach to debt. Flip open a textbook on economics and you’ll find something that looks like this:

\[
\max_{c^t_i} u^t_i(c^t_i) \quad \text{s.t.} \quad y^i = p_c c^t_i
\]  

(1)

There is an economic actor \( i \), who can be described as deriving ‘utility’ in *economese,* from choosing a quantity of \( c \) or consumption goods: \( u^t_i(c^t_i) \). The assumption is that if the choice is between \( c^i \) and \( c \) where \( c^i > c \), then \( c^i = c^i \) or \( c^i > c \), because \( u^t_i > u^i \) and higher utility is preferred to lower utility or \( u^t_i > u^i \). In calculus terms this is expressed as \( \delta u^i / \delta c^i > 0 \), where it is assumed that the behavior of the economic actor is equal to maximizing – hence the ‘max.’ – \( u^t_i(c^t_i) \) and because we’re dealing with a world of scarce resources, this is always subject to some constraint: here, \( y^i = p_c c^t_i \) where \( p_c \) is the price of \( c \) and \( y^i \) is the income of *Max U,* as baptized by McCloskey (see McCloskey 2002 p.23). This is the basic model used by economists to describe the consumption formalized in the calculus of variations. Solve it by maximizing \( u_t \) by choosing \( c^t_i \) consistent with the constraint \( y^i = p_c c^t_i \) and the result is what's known in economese as *equilibrium:*

\[
c^{t^*}_i = \frac{y^i}{p_c}
\]  

(2)
Where $c^*_i$ is the equilibrium-choice of Max U. The meaning of equilibrium is that is a:

“a constellation of selected interrelated variables so adjusted to one another that no inherent tendency to change prevails in the model which they constitute”

(Machlup 1958 in Chiang 1984 p.35)

Or, as Samuelson (1975 p.9) put it “[an] equilibrium is not something which is attained; it is something, if attained, displays certain properties”. In this particular case,

$$\frac{\delta c^*_i}{\delta y_i} > 0$$ (3)

$$\frac{\delta c^*_i}{\delta p_c} < 0$$ (4)

In economese: an increase in income, increases demand and an increase in price decreases quantity demanded. Some ‘schools’ consider this adjustment process disequilibrium; other ‘schools’ consider this adjustment process to take place in equilibrium (see for more Machlup 1958). The distinction here does not matter, as the adjustment-process is instantaneous (i.e. there is no process). This is a model without time and consequently a model without debt. Add in $t$, and we have a necessary and sufficient condition for the existence of debt. Flip a bit further through the textbook – probably until you see the words ‘Inter-temporal choice’ – and you’re bound to encounter something that looks somewhat like this:

Note the parallel with a mirror from the introduction. Both the mirror and equilibrium share the property that everything happens (practically) simultaneously.
All the variables are the same except that they’re now indexed with the subscript \( t \). This means that the time, that \( y_t \) is received, and that the time that \( c_t \) is chosen, now have some significance. Further there are two new variables: \( r \) and \( \beta^t \), where \( 0 < \beta^t < 1 \) and \( r \geq 0 \). \( r \) is the interest rate, that is the rate or price at which Max U can borrow and lend, and \( \beta^t \) is the result of the rate of time preference \( \rho^t \); the values chosen mean that Max U prefers to consume sooner rather than later: i.e. \( \rho^t > 0 \) and \( \beta^t = 1/(1 + \rho^t) \). The reason that the time at which income is received has some significance is that it determines how many units of \( c \) Max U can choose at that time and it is the utility per time period that now matters to Max U. Solving for \( c^t \), one can now derive how much Max U will lend or borrow at time \( t \) against rate \( r \), here denoted by \( B^t_t \): \[
B^t_t = p_c c^t - y_t
\] (6)

This itself will be a function of \( \rho, r, p_c \) and \( y_t \); changes in \( B^t_t \) are to be explained in terms of the aforementioned variables. A debt contract in the standard model is therefore something where \( B^t_t \) at time \( t \) is exchanged for \( B^t_t \) at time \( t + s \) within the above model this is equal to the promise of \( B^t_t(1 + r)^s \).

Debt in economese is the subset \( d^t_{econ} \) of \( D^t \) consisting of the elements \( \{c,y,p_c,u\} \) that are mapped onto one another to describe the behavior of Max U. “Mapping”, here refers to the correspondence of particular elements of the subsets of \( d^t_{econ} \) (i.e. the elements of that set) to the other elements of the subsets for reasons of theory. The difference with the second approach is thus, that whilst the elements of the set \( d^t_{econ} \) also remain the same as in the past, that here the functional relationship is the organizing principle, instead of whatever happened to be the case in
the past. The difference with the first approach is that this organizing principle allows us to select or create *de novo* the meaning of debt.

1.2 An economic concept of debt

Conspicuously absent above are two out of the three features of the real world that are of some importance to us\(^6\). The first is that income does not fall from the sky like manna from heaven; income has to be earned through the sweat of the brow, here conveniently denoted by \(e^i_t\) for effort. Second, is that whatever we become, part will be the result of hard work and part will be the result of luck; luck will be denoted by \(\varepsilon^i\) and is normally distributed with a mean \(\mu_i\) and variance \(\sigma_i^2\) or \(\varepsilon^i \sim N[\mu_i, \sigma_i^2]\). We can now write down the relationship between luck and effort and (nominal) income as followed:

\[
y^i_t = e^i_t + \varepsilon^i
\]  

(7)

The sweat of the brow or the cost of effort as so many things in life is assumed to be convex\(^7\) and can be written down as

\[
s(e^i_t) = e^i_t^2
\]  

(8)

with \(s\) for sweat. Before we re-write the maximization problem and define an economic concept of debt for our purposes, it is necessary to spend some time on the meaning of luck in this model.

---

\(^6\) The third feature is the relation to the creditor or intermediary; the relationship with the creditor or intermediary will follow later.

\(^7\) The cost of effort is convex meaning that cost of effort increases and increases further the more effort has been exerted (i.e. the idea that the increase from 10 – 20 km/h costs more than the increase from 0-10 km/h). This is such a pervasive feature of the real world that it scarcely requires further explanation.
1.2.1 The luck bit

An individual’s luck or $\varepsilon^I$ can be broken down into four parts:

- Shocks to the economy as a whole;
- Shocks to the price level;
- Shocks to the type of actor;
- Shocks to the actor as an individual.

The first two shocks are risks the actor shares with all other actors: shocks to the economy as a whole or $\varepsilon_A$ and shocks to the price level or $\varepsilon_P$. The other two shocks are specific to the actor or the group the actor is part of. The first pair is often referred to as *systemic* whilst the second pair is referred to as *idiosyncratic*.

**Shocks to the economy as a whole and shocks to the price level**

To illustrate the first two shocks, let’s imagine a two period economy inhabited by two actors: Milton and Paul. The economy produces apples and tokens each period, and each period these perish. Apples can only be purchased for tokens and tokens can only be used to purchase apples; the number of tokens is the product of the price and the number of apples. Paul and Milton each receive half the number of tokens, but they only care about apples. In the first period the economy produces 100 apples at a price of 1 each; in the second period the economy is expected to look the same. When Paul borrows 20 tokens from Milton the first period in exchange for the promise of 30 tokens the next period, they consume 70 apples and 30 apples respectively, and expect to consume 20 apples and 80 apples respectively. A price level shock is, when instead of 1 token, apples cost 2 tokens in the next period; the number of tokens is therefore 200, and Paul

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8 This is similar to the terminology used in the Capital Asset Pricing Model or CAPM for intimae. The CAPM formalizes the idea that, if risk can be diversified away by buying other assets, that is to say risk is *idiosyncratic*, then it carries no reward; if risk cannot be diversified away on the other hand, that is to say it shared characteristic of assets, it is *systemic* and then it carries a reward. For the first type of risk no one has to bear it, for the second type of risk someone has to bear it and receive a reward for it.
and Milton consume 35 apples and 65 apples respectively. A shock to the economy as a whole occurs when instead of 100 apples, there are now only 80 apples; the number of tokens is therefore 80, and Paul and Milton consume 10 apples and 70 apples respectively.

In the first scenario in terms of \( y_i^t = e_i^t + e^i \):

### First Scenario: Price-level shock

<table>
<thead>
<tr>
<th></th>
<th>Expects ( e_i^t )</th>
<th>Price-level shock ( e_p )</th>
<th>Total ( y_i^t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paul</td>
<td>50 tokens</td>
<td>50 tokens</td>
<td>100 tokens</td>
</tr>
<tr>
<td>Milton</td>
<td>50 tokens</td>
<td>50 tokens</td>
<td>100 tokens</td>
</tr>
</tbody>
</table>

In the second scenario:

### Second Scenario: Shock to the Economy as a whole

<table>
<thead>
<tr>
<th></th>
<th>Expects ( e_i^t )</th>
<th>Aggregate shock ( e_A )</th>
<th>Total ( y_i^t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paul</td>
<td>50 tokens</td>
<td>&lt;10 tokens&gt;</td>
<td>40 tokens</td>
</tr>
<tr>
<td>Milton</td>
<td>50 tokens</td>
<td>&lt;10 tokens&gt;</td>
<td>40 tokens</td>
</tr>
</tbody>
</table>

There are two things to note in this example. First, is that if Milton would insist that Paul would pay 40 tokens corrected for the increase or decrease in the price-level, then Milton would ‘always’ consume 80 apples the next period. Second, is that if the number of tokens would determine the price-level – the price-level is the quotient of the number of tokens and apples – and if the number of tokens would remain constant, then Paul and Milton would always expect to consume 20% and 80% of the total number of apples in the second period.
For our purposes it will be assumed that the number of tokens remains constant and the monetary authority thus can be said to engage in a form of NGDP-targeting. According to Dormian & Eagle (2005) this is Pareto-efficient, provided Paul and Milton have the same constant relative risk-aversion, as the risk for shocks to the economy as a whole would be shared equally between the two of them. What this assumption therefore amounts to, is that, when the suppliers of loanable funds and the demanders of loanable funds have the same risk-aversion, then no terms need to be discussed for the sharing of the risk of price-level shocks, and shocks to the economy as a whole. We can therefore exclude $\varepsilon_A$ and $\varepsilon_D$ from our discussion.

*Shocks to the actor type and to the actor individually*

From above we know that all the actors are exactly alike, down to their cost of effort. The only way in which these actors therefore can differ from one another is plain luck or $\varepsilon^i$. Let’s assume that the entire economy consists of $n$ actors or $i = 1, 2, \ldots, n$, and let’s further assume that for each actor $i$ there are $m - 1$ actors denoted by $j$ such that ex post:

$$\varepsilon^i \equiv \varepsilon^j \quad (9)$$

---

9 Nominal GDP-targeting is – briefly stated – the idea that if $MV \equiv PY$, then the monetary authority should keep $MV$ so as to keep $PY = NGDP$ constant, or on a linear trend by having $MV$ follow an upward path. If the income shares remain the same, then each individual’s share of output, or RGDP, should also remain the same. This idea – NGDPLT – has since the start of the recession gained in currency due to – mainly – the advocacy of S.B. Sumner, a professor of economics at Bentley University (USA) and proprietor of the blog TheMoneyIllusion; for he argued that the cause of the recession in the USA could be traced back to a deviation from the 5%-6%-year trend of the Great Moderation. The idea itself has considerable intellectual pedigree, as various authors, to as far back as at least the 19th century, can be read as having advocated it. It is beyond the scope of this thesis to go into this, apart from noting, that the argument depends on the monetary policy that determines the value of the unit of account. A straightforward extension of this thesis would be to relax this assumption and generalize the argument across monetary policy rules.

10 If two actors are otherwise identical, then it is their risk-aversion that determines their willingness to bear risk. Here, both are equally willing or unwilling to bear risk.
To always receive the same shock to income, is what it means for an actor to be of the same type or $\theta$ as another actor. It is assumed that actors know their type. Finally,

$$\sum_{\theta=1}^{n/m} e^{\theta} \equiv 0$$  \hspace{1cm} (10)

or the shocks to the types, is assumed always to sum to zero.

The first assumption captures the idea that in any economy a division of labor can be found as a result of a comparative advantage. Some people make lousy painters, but are even worse carpenters whilst others are good painters, but even better carpenters; that society will therefore consist of lousy painters and great carpenters. For example, if Alfred can paint two fences in the time that it takes him to build one, and Neville can paint three fences in the time that it takes him to build six, then when Neville is painting one fence he could have built two instead; Alfred on the other hand would only have managed to build half a fence. Clearly, Neville should build fences and Alfred should paint these.

The second assumption captures the idea that Alfred will suffer when the painting industry is hurt, and Neville will not be too happy when carpentry suffers a downturn. That actors know their type simply means that painters know they’re painters and carpenters know they’re carpenters: both Alfred and Neville can tell the difference between a brush and a hammer.

And finally, that the shocks to nominal income sum to zero, means that when Alfred gains, Neville loses and vice-versa: sometimes people want their fence repainted and sometimes they only need a new fence. That the shocks to income sum to zero therefore expresses the idea that this is not a shock to the economy as a whole or a price-level shock, but rather a re-allocation within the economy; when aggregate nominal income or NGDP is kept constant then shocks have to sum to zero.
All in all, what the above states is that there is no distinction between these two types of shocks apart that from the perspective of the individual $\varepsilon_i$ is observed and from the perspective of society or in aggregate $\varepsilon_\theta$ is observed.

1.2.2 The economic concept of debt

To the economic concept of debt above, which consist of the elements $\{c, y, p_c, u\}$, we’ve now added the elements, effort, and the cost of effort, and risk, or $\{e, s, \varepsilon\}$. To write down the problem one additional simplification will be made: it will be assumed that the actor does not earn any income in the first period and therefore has to borrow for consumption. The demand for loans, is therefore, defined as the demand for consumption in the first period.

$$B_1^* \equiv c_1^*$$ (11)

Where we assume that $p_c = 1$. As the economy only exists for two periods, we can write in the actor’s problem, that consumption in the second period, is equal to expected income minus the debt that has to be repaid:

$$c_2^* = E[y_2^*] - c_1^*(1 + r)$$ (12)

The stars above the variables denote that these variables are equilibrium values; the actor already knows the choices made in equilibrium, when the choice is made how much to consume in the first period. The second period consumption is obvious: expected income in the second period is an equilibrium choice, because the actor anticipates how much effort will be exerted then. The

---

11 This is primarily to reduce the number of parameters in our model: we’re not really interested how the demand for loans changes with price.

12 $D_1^* = B_1^*(1 + r)$
actor lives according to the Greek maxim γνῶθι σεαυτόν: know thyself. To illustrate: if Simon knows that he will not work on a project due the next day when in the evening the European Football Championship is on, then he will work more during the day\textsuperscript{13}.

For an actor with an exponential utility function exhibiting Constant Absolute Risk Aversion or CARA, the problem can be written down as followed:

$$\max_{c_1} E u \left[ -\exp\left[ -a^i c_1^i \right] + \beta_1^i \left( -\exp\left[ -a^i \left( E[y_2^i] - D_1^i - s(e_2^i) \right) \right] \right) \right]$$  \hspace{1cm} (13)

The use of a CARA-utility function is not without its problems, as it has some unrealistic implications when it comes to the behavior of the actor concerning wealth and risk-aversion. The advantage of using this utility function, however, is that the actor explicitly solves the problem with distribution of the shocks in mind for\textsuperscript{14}:

$$\exp[-a\varepsilon] = \exp\left[-a\mu + \frac{a^2}{2}\sigma^2\right]$$  \hspace{1cm} (14)

Where, $\varepsilon \sim [\mu, \sigma^2]$, and $a$ is the preference towards risk of the actor. $a$, here, is larger than zero and the further $a$ is away from zero the more risk-averse the actor is. In our case this means that when we solve for the demand for loans that this is a function of $\sigma_i^2$ and $\mu_i$:

$$B_1^{i*} = \frac{a^i \left( e_2^{i*} - e_2^{i*2} + \mu_i - \frac{1}{2} a^i \sigma_i^2 \right) + \log \left[ \frac{1 + \rho_i}{1 + r} \right]}{a^i(2 + r)}$$  \hspace{1cm} (15)

\textsuperscript{13} It is of course debatable whether this amount of wisdom is reasonable as an approximation.

\textsuperscript{14} \int_{-\infty}^{\infty} \exp[-ae] f(\varepsilon) d\varepsilon where $\varepsilon \sim [\mu, \sigma^2]$ and $f(\varepsilon)$ is the normal distribution; the definition of an expected value is $E[x] = \int_{-\infty}^{\infty} xf(\varepsilon) d\varepsilon$, or the integral of every possible outcome of $x$ multiplied by its probability of occurrence.
Where the amount of debt can be written as:

\[ D_1^* = B_1^* (1 + r) \]  \hspace{1cm} (16)

Debt is now defined as a contract, where, in addition to the time preference and the rate of interest, perceived risk and anticipated effort play a role in the decision to borrow. This definition captures the common-sense observation that when you are more impatient, are more certain about the future, or when you anticipate to earn more, then you’re also willing to borrow more for the same price; whilst if the rate of interest increases you borrow less. The first can be described, as what’s commonly called an outward shift of the demand-curve for loans whilst the second is a movement along that demand-curve. The amount of debt you owe, therefore, reflects your impatience and your reliance on your expectations about the future. Problems with indebtedness are according to our concept of debt, therefore, part due to impatience, part due to the wrong expectations, and part due to over-reliance by the borrower; the other half of the problem would be the patience, the wrong expectations, or over-reliance by the lender\(^{15}\).

1.3 A convenient way to describe debt contracts

One way to discuss debt is to discuss it in terms of the unit of account, or in terms of what can be referred to as its \textit{absolute magnitude}; we did this above. This however, is not a very intuitive way to continue the discussion. A better and more intuitive way to discuss debt would be as a fraction of income or its \textit{relative magnitude}. To illustrate, if both Carmen and Kenneth, are each

\(^{15}\) See also Graeber (2011) p.2-4; Graeber stresses the other side of the relationship and the perverse effects of when debt is ‘guaranteed’ to be repaid. In the same vein, see also \textit{Hadley v. Baxendale} [1854] EWHC J70 in Cooter & Ulen (2012) p.335-337.
9,000 in debt that is due next year, we know how much they owe in terms of the unit of account, but we have no idea as to whether either can repay the amount owed; if Carmen earns € 100,000 a year and Kenneth barely scraps by with € 10,000 a year, then for Carmen we see that the debt at 9% of income is manageable, whilst in Kenneth’s case, we see that at 90% of income this is a bit much. Furthermore by discussing the relative magnitude we can discuss debt in a way that makes our conclusions applicable to actors who differ in income as much Carmen and Kenneth do. We therefore write now,

$$\kappa_i = \frac{D_i^*}{e_i^* + e^i}$$

(17)

where \( \kappa_i \) expresses debt as that share of income to be used to repay debt ex post or after \( e^i \).

1.3.1 Risk-sharing in debt contracts

If we write \( \kappa_i \) as a function of the size of the shock to income or \( \kappa_i(e^i) \) we can distinguish four different debt contracts based on how risk is allocated between the parties involved:

I. The debt to be repaid is a fixed fraction of income or \( \kappa_i(e^i) = \bar{\kappa}_i \);

II. The debt to be repaid is fixed in absolute terms or \( \kappa_i(e^i) \neq \bar{\kappa}_i \);

III. The debt to be repaid is the minimum of some fraction of income or fixed in absolute terms or \( \kappa_i(e^i) = \min[\kappa_{\min}(e^i)y_2^*(e^i), B_1^*(1 + r)]/y_2^*(e^i) \);

IV. The debt to be repaid is the maximum of some fraction of income or fixed in absolute terms or \( \kappa_i(e^i) = \max[\kappa_i(e^i)y_2^*(e^i), B_1^*(1 + r)]/y_2^*(e^i) \).

16 Whilst the actors in our model do not differ with respect to income ex ante, they do differ with respect to income ex post when they’ve had their share of luck.
The interpretation given to $\kappa_i(\epsilon^i)$, is that it represents those terms of the debt contract which deal with the allocation of risk. Within these four types it is possible to make a second division. For in the case of the first two types (“type I” and “type II”), one party is exposed to both ‘good’ and ‘bad’ shocks, whilst in the case of the second two types (“type III” and “type IV”), one party is exposed to the ‘good’ shocks, whilst the other party is exposed to the ‘bad’ shocks.

1.3.2 Some brief examples of risk-sharing in contracts

In order to illustrate what it means to say that $\kappa_i(\epsilon^i)$ behaves in a particular way, we will discuss some examples below.

---

1. $\kappa_i(\epsilon^i)$ is analogous to those contractual doctrines that deal with the efficient allocation of risk between parties, such as frustration of purpose and impossibility (see also Cooter & Ulen 2012 p.349-352).
Type I contracts: fixed in relative terms

When a contract is fixed in relative terms, it means that the debt is always the same fraction of income or:

\[ D_{1t}^* = \bar{K}_t y_{2t}^* (\varepsilon_t) \]  \hspace{1cm} (18)

The most common example of such a contract, is one where the sole shocks that affect income are shocks to the price-level or \( \varepsilon^i = \varepsilon_p \) and where the contract is indexed for inflation. The logic is then, that because the individual’s share of NGDP remains the same, the quotient of the price-level in the second period and the first period must always equal the quotient of the expected nominal income and the realized nominal income or:

\[ \frac{P_{2t}}{P_{1t}} \equiv \frac{y_{2t}^* (\varepsilon_p)}{E[y_{2t}^* (\varepsilon_p)]} \]  \hspace{1cm} (19)

And therefore,

\[ \bar{K}_t = \frac{D_{1t}^* (1 + \frac{P_{2t}}{P_{1t}})}{y_{2t}^* (\varepsilon_p)} \]  \hspace{1cm} (20)

This is the same result as the result that Milton would have had, if he had his way in section 1.2.1.

Another example is a non-voting stock in a corporation. The stock is a fraction of the total value of the corporation and the total value of the stock can be said to be determined by its earnings. The difference with the contract indexed for inflation is that the real value of the loan would not remain constant, but fluctuate with the corporation’s fortune. A further difference
between this contract and most debt contracts is that a stock does not have to be ‘repaid’ by the issuer at some future date\(^{18}\); this is a characteristic that contract shares with a *consol*: a consol is a form of a perpetual bond with a difference that it can be re-purchased (see Mishkin 2004 p.67).

Finally, if one is willing to take a more metaphorical view of debt than the one defined here earlier, then one could also include a proportional income tax or a flat tax under this heading.

*Type II contracts: fixed in absolute terms*

In contrast to the above, in a debt contract where the payment is fixed in absolute terms the amount to repay will vary as a fraction of income, and from the debtor’s perspective: often decreasing in good times and increasing in bad times. This relationship can be written down as:

\[
k_i(\varepsilon^i) = \frac{D_i^*}{y^*_2(\varepsilon^i)}
\]

where, \(\varepsilon^i\) is the individual or type shock to the income of the debtor. The literature identifies this type as the most important contract used in the financing of various activities\(^{19}\). The reason is that the terms within most bank loans take this form and bank loans are by far the largest source of finance for non-financial businesses; anyone who has ever taken out a loan will probably be familiar with this type of contract. The presumption is therefore, that this is the efficient contract, and a large part of the optimal debt literature could be described as concerned with explaining why this could be efficient (see for example Freixat & Rochet 1999 Ch. 4 p.91-133; the authors here explore the relationship between lenders and borrowers where the lender is a financial intermediary by going over some of the more persuasive models in the literature). The agreed on

\(^{18}\) Another difference is that it is subordinated to all other debt contracts in insolvency.

\(^{19}\) Over the period 1970-1996 bank loans accounted for ~ 40% of finance in the USA, for 80% in Germany and for 90% in Japan (see Mishkin 2004 p.171).
terms of those contracts, however, are limited by bankruptcy legislation and, if we read that legislation as writing terms into every contract, then it is the contract of the third type that is most common (see also Freixtas & Rochet 1999 p.94; the authors observe that risk-sharing whilst optimal, scarcely takes place at all and that when limited liability is taken into account contracts take the form of the third type). One could therefore argue that, nor the contract of the first type, nor the contract of the second type, are contracts that actually exist. Just as there is no such thing as a perfect square; these contracts primarily serve the role of ideal-types. Finally, if the contract of the first type can be seen as a proportional tax, then the repayment schedule of this contract is very much like a lump-sum tax ex ante and thus does not affect incentives\textsuperscript{20}.

Type III contracts: ceilings on $\kappa_i(e^i)$

As mentioned above, when bankruptcy legislation is taken into account, then debt contracts of the third type are the most common contracts. These contracts can be described as putting a ‘ceiling’ on $\kappa_i(e^i)$ for when graphed as in figure 1.3.2.c these contracts show a distinct ‘kink’:

The reason for this ‘kink’ is that those contracts do not allow income, net of debt, to be reduced below a certain minimum due to bad luck. How much bad luck someone can have, or what the minimum income is, is indicated in the graph by the dashed line.

\textsuperscript{20} One example is the unsuccessful Community Charge implemented under Thatcher; it goes without saying that this similarity does not always make these contracts popular.
Mathematically we can write that:

\[
\kappa_i(\varepsilon^i) = \frac{\min\left[\kappa_{min}(\varepsilon^i)\gamma^*_2(\varepsilon^i)B^*_1(1+r)\right]}{y^*_2(\varepsilon^i)} \tag{22}
\]

The minimum income or \(y_{min}^*\) – note that it is invariant with respect to luck, effort or the identity of the actor – is there defined as:

\[
y^*_2(\varepsilon^i) - \kappa_{min}(\varepsilon^i)y^*_2(\varepsilon^i) \geq y_{min}^* \tag{23}
\]

Or,

\[
\kappa_{min}(\varepsilon^i) = \frac{y^*_2(\varepsilon^i) - y_{min}^*}{y^*_2(\varepsilon^i)} \tag{24}
\]

where of course \(\kappa_{min}(\varepsilon^i) \geq 0\). This is the limited liability constraint in the contract; the liability to repay can never be so large that income ex post drops below a certain number that is larger or equal to zero. This however also means that, the opportunity cost of the amount borrowed is always larger than value of expected value the debt contract ex ante or:

\[
B^*_1(1+r) \geq \int_{-\infty}^{\infty} \kappa_i(\varepsilon^i)\gamma^*_2(\varepsilon^i)f_n(\varepsilon^i)d\varepsilon^i \tag{25}
\]

The integral is the expected value of the contract. The first two terms of the integral show how the ex post value of the contract – the pay-off – changes with changes in \(\varepsilon^i\), whilst the last term – the density function – is the probability of a particular \(\varepsilon^i\) occurring. Add up the product of all these
terms for all possible $e^i$, and you have the expected value of the contract. Graphically, the difference in pay-offs multiplied by their probability as a function of $e^i$, can be represented in this way:

Figure 1.3.2.e: The shaded area represents the difference in expected value between a contract of type I and type III.

The area is a function of variance, and a substantial variance – here income is equal to one standard deviation – is necessary to make the area this large in proportion. Further implicit in this picture is that second period income cannot become lower than zero.

To deal with this problem, so that the expected value ex ante is equal to the amount borrowed, the contract can contain one or more of three strategies. First, is a premium upfront equal to the difference; second, is to add a premium to what the opportunity cost of capital would be, if there was no limited liability. Third, is to limit the amount borrowed so as to make the difference effectively equal to zero. The first two strategies are of course equivalent and in practice are used in combination with the third strategy\(^\text{21}\). All three have in common that the amount borrowed is decreased voluntarily or involuntarily, and the first two shift the burden from those with bad luck to those with good luck. Graphically,

\(^{21}\) The reason is that raising the premium exacerbates the problem of adverse selection. However see also Stiglitz & Weiss (1981) p.408-409: “Under those circumstances credit restrictions take the form of limiting the number of loans the bank will make, rather than limiting the size of each loan, or making the interest rate charged an increasing function of the magnitude of the loan, (..)”
Figure 1.3.2: the shaded area represents the difference in the pay-offs multiplied by the probability between a contract of type I and a contract of type III when a premium is included and the amount borrowed is decreased. The goal is to make the area above equal to the area to the left.

To continue the analogy with debt contracts and taxation: this contract is equivalent to a progressive rate that makes all income equal up to the dashed line, after it is equivalent to a lump-sum tax. It is this first part of the rate structure that causes some problems when it comes to the effort exerted in equilibrium. If effort is exerted before actors know their luck, then effort is a single value; otherwise effort is a function of the shock to income. This – moral hazard – is one of the two problems that explain why debt contracts often contain many complicated provisions concerning what debtors can do or can’t do and why something such as collateral exists; the other problem is adverse selection (see Mishkin 2004 pp. 174-186 for a general explanation; see Freixtas & Rochet 1999 for in depth explanations of the various problems and solutions). \(^{22}\)

Two common and two uncommon real world examples of contracts with a ceiling on \(\kappa_i\) are:

\(^{22}\) This problem is not unique to debt contracts: see for example Baker (1992) where after the wage contract is signed a shock is observed by the employee, but not by the employer, and an incentive scheme has to be agreed upon ex ante where pay is based on a proxy for performance.

\(^{23}\) This thesis, because of the question, is more generally concerned with the terms of a contract to deal with the behavior of the debtor after the contract is signed, rather than with the problems of selecting debtors; in the model discussed all actors are alike ex ante.
• Loans made in the course and for the purpose of a college education (e.g. tuition, living expenses); in the Netherlands for example, the amount to repay (monthly) is contingent on (monthly) income and limited in time\(^{24}\).

• Loans made to individuals in general, who, due to a shock to income are not able to repay without dropping below some existential minimum; under the Dutch law of personal bankruptcy, a maximum amount is reserved out of the debtor’s income to pay off debts for a number of years; after this period, it is no longer possible to enforce whatever remains to be repaid of the debt in court\(^{25}\).

• Debts which due to priority are subordinated to other debts; creditors who take security in essence set \(y_{min}^{*}\), where now \(E[y_{2}^{*}] = A^i + e_{2}^{*} + e_{1}\) where \(A^i\) is an asset owned by the debtor that is used as a form of collateral\(^{26}\).

• Debts so small that the (marginal) collection costs are not outweighed by the (marginal) benefits of collecting.

*Type IV contracts: floors under \(\kappa_i(e^1)\)*

If the contract above can be said to put a ‘ceiling’ on \(\kappa_i(e^1)\), then this contract can be described as putting a ‘floor’ underneath it. Here, debt drops as a proportion of income until income hits a certain number after which the debt to be repaid rises in proportion with income. As with the contract of the third type, effort can now become a function of the shock to income. Mathematically we can write that:

---

\(^{24}\) Art. 6.1 – 6.17 Wet Studiefinanciering 2000
\(^{25}\) Wet schuldsanering natuurlijke personen 1998; see in particular Art. 295-4 Faillisementswet.
\(^{26}\) However see also Lacker (1991); Lacker explains part of the ubiquity of the standard debt contract as a result of the *implicit or explicit* presence of collateral for loans. The logic of collateral is very much the same as that of a hostage in Williamson (1983). Here, however, we will not deal with the effects of collateral on the behavior under debt.
One example of a contract that behaves in this way is the convertible bond. In the case of the convertible bond, the contract is convertible into equity, contingent on the share price, where the share price can be said to be a function of income. Similar to above, we can depict the pay-offs multiplied by their probabilities:

\[ \kappa_i(\epsilon^l) = \frac{\max[k_i(\epsilon^l) y_2^{*}(\epsilon^l), B_1^{*} + l(1 + r)]}{y_2^{*}(\epsilon^l)} \]  

(26)

The difference with the third type of contract is that, here, a premium is subtracted and the debt is increased. In this the contract of the fourth type is exactly the reverse of a contract of the third type.

### 1.3.3 Can all debt contracts be described this way?

What this framework illustrates is that a number of contracts can be described by how the fortune of the debtor affects the fortune of the creditor. This raises the question whether all contracts can be described in this way. The answer to this question is likely to be yes as the logic is
no different from creating new financial products through the use of other financial contracts, most notably through the use of option contracts\textsuperscript{27}. This thesis is an application of that principle and by using this framework we can now describe something – the right to buy-back pre-mature debt – we do not know in terms of what we do know.

\textsuperscript{27}In Cooter & Ulen 2012 p.313 fn7 it is shown that a contract with \textit{perfect opportunity cost damages} is equivalent to an option contract when the goods in question are close substitutes.
Chapter 2: An optimal debt contract

“The only man who sticks closer to you in adversity than a friend is a creditor.”

-Unknown

2.1 The optimal debt contract is a linear contract

In order to deduce the optimal debt contract it is important to first ask what it could possibly look like. Here, it is assumed that the optimal debt contract is a linear contract. Linear means that the contract consists of a fixed part and a flexible part where the flexible part is proportional to luck or income. The optimal debt contract is thus a combination of the first pair of contracts: type I and type II.

There are two reasons why this is a plausible assumption to make for this purpose. The first reason is that risk-aversion is symmetric; a risk-averse actor does not care about good luck or bad luck but only cares about fortune itself. Therefore when difference in risk-aversion is the reason for insurance then the actor should insure against luck whether it be good or bad by transferring it to someone who cares less about the luck.

To illustrate: Gary faces an uncertain future for without insurance Gary can expect to earn €100. There are however three possible outcomes, each equally likely: in the first outcome Gary earns €150 in the second outcome €50 and in the third outcome €100. ‘Bad luck’ here means that Gary earns €50 instead of the expected €100, similarly ‘good luck’ means that Gary earns €150. Whether luck is good or bad means whether the outcome is better or worse than expected: this is the most common usage. If he insures against bad luck, then he can expect to earn €125 before the insurance premium. Bad luck now would be that Gary earns €100. This example illustrates that, for one it is not possible to consistently only care about good luck or bad luck, and two, that the only way to insure against bad luck or good luck is to insure against all possible outcomes. The
same intuition also applies to contracts that put more weight on particular outcomes for why not put more weight on all other outcomes as well?

Mathematically this intuition is captured by writing down the utility-function of a risk-averse actor as a concave function of the possible outcomes. The result is that the utility of the expected outcome is higher than the expected utility of the outcome (see also Varian 2010 p.177). A linear contract in the model will therefore dominate any contract that is limited to only a part of the outcomes.

The second reason is that the difference between a symmetric and asymmetric contract is small when the variance is low and that the problem of default does not undercut much of the logic of the argument; the advantage is that it makes the problem much easier to manage\textsuperscript{28}. Further when variance is high and default becomes a serious problem we will usually be dealing with a whole different set of contracts where other problems will dominate the discussion\textsuperscript{29}.

2.1.1 The linear debt contract

Formally, we can write down two equations that have to hold ex ante (see here also Gibbons & Waldman 1999)\textsuperscript{30}. First, for the linear contract when income is insured:

\[
E[D_1^{1*}] = f + \kappa_i E[y_2^{1*}(\varepsilon^1)]
\]

(27)

And second that what is contracted for:

\textsuperscript{28} An apt comparison here would be the Fisher equation for nominal rates or \(1 + i = (1 + r)(1 + \pi)\); the linear approximation \(i \approx \pi + r\) can be used for low values of the variables for \(\pi r \approx 0\).

\textsuperscript{29} See for example the description of the venture capital contracting model in Gilson (2003)

\textsuperscript{30} The model used in this section is loosely based on that paper (e.g. modeling of the contract, contractibility of effort, choice of utility function).
Because these equations are about the same expected amount ex ante, we can solve for $f$ by re-arranging these into:

$$ f = B_1^i (1 + r) - \overline{\kappa}_i \mathbb{E}[y_2^i(*)] $$ \hspace{1cm} (29)$$

And because $f$ has to be fixed ex ante we can drop the expectations operator:

$$ f = B_1^i (1 + r) - \overline{\kappa}_i e_2^i + \overline{\kappa}_i \mu_i $$ \hspace{1cm} (30)$$

The interpretation of $\mu_i$ here is that part of luck that systematically favors or hurts the debtor. This equation states thus that the fixed part of the contract is equal to the part borrowed minus that fraction of income that is insured. From the debtor’s perspective this should be equal to zero.

From an ex ante perspective we can therefore write for when a fraction of income is insured:

$$ E \left[ D_1^{iY^*} \right] = B_1^i (1 + r) - \overline{\kappa}_i e_2^i + \overline{\kappa}_i \mu_i + \overline{\kappa}_i \mathbb{E}[y_2^i(*)] $$ \hspace{1cm} (31)$$

For when the contract can be made contingent on only luck we can write instead:

$$ E \left[ D_1^{i\epsilon^*} \right] = B_1^i (1 + r) + \overline{\kappa}_i \epsilon^i - \overline{\kappa}_i \mu_i $$ \hspace{1cm} (32)$$

This result can be obtained when luck is substituted for income above.
When neither is contractible, then the linear contract and the amount for which is borrowed are one and the same:

\[ E[D_t^{i0}] = B^*_1(1 + r) \]  

(33)

\( \bar{\kappa}_i = 0 \) and the contract contains no terms to share the risk between debtor and creditor.

### 2.2 The optimal debt contract with symmetric information ex ante

To derive the optimal debt contract three problems need to be solved simultaneously:

- The problem of the borrower;
- The problem of the intermediary;
- The problem of the lender.

For the problem of the lender the reader is referred back to section 1.2.1; the contract between the lender and the intermediary can be seen as the problem of the optimal contract between debtors and creditors without an intermediary concerning the value of the monetary unit. Based on the assumption in section 1.2.1, the value of the monetary unit is already optimal. The reason why the intermediary can be ignored is that the asset-side of the intermediary is perfectly diversified; this is a common assumption to make (see Freixtas & Rochet 1999 p.94) and is effectively equal to the assumption of a single intermediary which will be made below. The problem of the lender therefore has already been solved.

### 2.2.1 The problem of the intermediary

31 The problems of the actors were solved by use of Mathematica 8.0; the code is added separately to the thesis.

32 This is also consistent with how the relationship between intermediary and lender is structured in most countries, see for example the case of Foley v. Hill (1848) 2 HLC 28.
For the problem of the intermediary, for purposes of simplification, we will assume that there is only one intermediary; this is an innocent simplification as the argument does not depend on the competition for borrowers between intermediaries.

The problem of the intermediary is to maximize the volume of loans multiplied by the difference between the rate charged to the borrowers \( r \) and the rate paid to the lenders \( r_c \).

\[
\max_{r,r_c,k_t} \pi B_1^{k_t}(r - r_c) \tag{34}
\]

The intermediary maximizes profit by choosing the rate charged to the borrower and the terms of the contract \( k_t \). From the shape of this profit function, it is clear that both parties to the contract will choose the same value for \( k_t \). The reason is that the demand for loans and the welfare of the borrower decrease with variance and increase with (expected) income; the intermediary is fully diversified whilst the borrower is not, therefore the latter cares about variance whilst the former does not. Both therefore care about decreasing the variance for the borrower as this maximizes the expected income and the demand for loans; and as \( k_t \) trades-off incentives to earn income against insurance, both parties will choose the same \( k_t \). The logic here is the same as in Cooter & Ulen (2012 p.365/366) and a special case of the perfect contract construct: the non-price terms of the contract maximize the total gains from trade, whilst the terms concerning the price are used to divide those gains\(^{33}\). This leaves us with the problem of the borrower.

2.2.2 The problem of the borrower and the optimal debt contract

\(^{33}\)“The nonprice terms of a contract typically create incentives that affect the size of the surplus from exchange, and efficient nonprice terms maximize the surplus from exchange and thereby maximize its [the monopolist’s] profits. In contrast, the price terms typically distribute the surplus between the parties.” (Cooter & Ulen 2012 p.365) . The authors go on to state that this is not always the case; the problem there however lies not in bargaining between the parties but in the ignorance of one of the parties.
When the economic concept of debt was explained in section 1.2.2, we wrote for the borrower that:

\[
\max_{c_t} Eu \left[ -\exp\left[-\alpha^t c^t_1\right] + \beta_t^i \left( -\exp\left[-\alpha^t \left( E[y^t_{y^t}] - D^i_t - s(e^t_i)\right)\right]\right] \] (35)

In order to derive the optimal debt contract, the first step is to consider \( E[y^t_{y^t}] - E[D^i_t] \) instead of \( E[y^t_{y^t}] - D^i_t \) and re-write the problem of the borrower for each of the three contracts. The interpretation of this re-write is that instead of a non-contingent contract we now consider a contract that depends on what happens in the future. The solution to the problem, as in section 1.2.2, is the optimal consumption in the first period which by definition here is equal to the demand for loans. The second step is to solve for the amount of effort the debtor expects to exert under the contracts. Finally, both the first and the second solution will be substituted back into the original problem; the result is that all the variables are a function of the terms of the contract or \( \bar{\kappa}_t \). We can then solve for the optimal terms by maximizing the utility of the debtor.

**Step I: The demand for loans**

Recall from above that the contract for when income is insured was:

\[
E\left[D^t_{y^t}\right] = B^t_1(1 + r) - \bar{\kappa}_t e^t_2 - \bar{\kappa}_t \mu_i + \bar{\kappa}_t E[y^t_{y^t}] \] (36)

When we substitute this into \( E[y^t_{y^t}] - E[D^t_{y^t}] \) we can write:

\[
E[y^t_{y^t}] - E\left[D^t_{y^t}\right] = (1 - \bar{\kappa}_t)E[y^t_{y^t}] - B^t_1(1 + r) + \bar{\kappa}_t e^t_2 + \bar{\kappa}_t \mu_i \] (37)
And because $E[y_{2}^{i*}] = e_{2}^{i*} + \mu_{i} + E[\varepsilon^{i} - \mu_{i}]$, where $E[\varepsilon^{i} - \mu_{i}] = E[\varepsilon_{\mu=0}^{i}]$, the expectation of the borrower is:

$$E[y_{2}^{i*}] - E[D_{1}^{i*}] = e_{2}^{i*} + \mu_{i} - B_{1}^{i*}(1 + r) + (1 - \tilde{\kappa}_{i})E[\varepsilon_{\mu=0}^{i}]$$  \hspace{1cm} (38)

This is equivalent to the expectation of the borrower for when only luck is insured. The interpretation is that the debtor in the model can predict effort, but cannot predict luck. Finally, we can write that:

$$E[y_{2}^{i*}] - D_{1}^{i} = e_{2}^{i*} + \mu_{i} - B_{1}^{i*}(1 + r) + (1 - \tilde{\kappa}_{i})\varepsilon_{\mu=0}^{i}$$  \hspace{1cm} (39)

To derive the demand for loans, we use (see section 1.2.2) that:

$$\exp[-a^{i}\varepsilon_{\mu=0}^{i}] = \exp[-a^{i}\mu_{\mu=0} + \frac{a^{i2}}{2}\sigma^{2}_{l}]$$  \hspace{1cm} (40)

And through substitution of $a^{i}$ by $a^{i}(1 - \tilde{\kappa}_{i})$, the right-hand side of the equation is now:

$$\exp[-a^{i}(1 - \tilde{\kappa}_{i})\varepsilon_{\mu=0}^{i}] = \exp\left[\frac{(1 - \tilde{\kappa}_{i})a^{i}}{2}\sigma^{2}_{l}\right]$$  \hspace{1cm} (41)

And thus we arrive at the demand for loans:

---

34 This is equivalent to taking the integral of $\int_{-\infty}^{\infty} \exp[-a^{i}(1 - \tilde{\kappa}_{i})\varepsilon_{\mu=0}^{i}]f(\varepsilon_{\mu=0}^{i})d\varepsilon_{\mu=0}^{i}$ where $\varepsilon_{\mu=0}^{i} \sim N[0, \sigma^{2}_{l}]$ and $f(\varepsilon_{\mu=0}^{i})$ is the distribution of $\varepsilon_{\mu=0}^{i}$. For the solution is:

$$\exp\left[-\frac{1}{2}a^{i}(1 - \tilde{\kappa}_{i})(-a^{i}(1 - \tilde{\kappa}_{i})\sigma^{2}_{l})\right] \frac{1}{\sqrt{\pi}\sigma^{l}}$$ where the last part of the equation is equal to 1.
For \( \bar{\kappa}_t = 0 \) or the contract without insurance:

\[
B^*_1 = \frac{a^t \left( e^i_2 - e^{i+2}_2 + \mu_i - \frac{1}{2} a^i \sigma^2_i (\bar{\kappa}_t - 1)^2 \right) + \log \left[ \frac{1 + \rho^i}{1 + r} \right]}{a^i (2 + r)}
\]  

(42)

This is the same as in section 1.2.2.

**Step II: The expected effort**

The level of effort of the debtor is the result of maximizing the second part of the utility function above; the debtor observes \( \varepsilon^i_{\mu=0} \) and then decides how much effort \( e^i_2 \) to exert. We can therefore write:

\[
\max_{e^i_2} u_2 \exp \left[ -a^i \left( e^i_2 + \varepsilon^i - D^i - s(e^i_2) \right) \right]
\]  

(44)

When income is insured this is:

\[
\max_{e^i_2} u_2 \exp \left[ -a^i \left( e^i_2 + \varepsilon^i - f - \bar{\kappa}_i e^i_2 - \bar{\kappa}_i \varepsilon^i - s(e^i_2) \right) \right]
\]  

(45)

where \( f = B^*_1 (1 + r) - \bar{\kappa}_i e^i_2 - \bar{\kappa}_i \mu_i \) is the fixed sum due no matter what happens: it’s a sunk cost. The optimal effort of the borrower is now
where the marginal benefit of effort equals the marginal cost of effort. This is equal to:

$$e^{y^*}_2 = \frac{1 - \bar{\kappa}_i}{2}$$  \hspace{1cm} (47)

Similarly, both for when only luck is insured and when neither income nor luck is insured, the optimal effort is:

$$e_2^e = e_2^{0+} = \frac{1}{2}$$  \hspace{1cm} (48)

**Step III: The Optimal Debt Contract**

Now that we’ve solved for the demand for loans and the equilibrium level of effort by the debtor, we can substitute these back into the original problem below and solve for $\bar{\kappa}_i$:

$$\max_{\bar{\kappa}_i} \left[ - \exp[-a_i c_i] + \beta_i \left( - \exp \left[ -a_i \left( E[y^*_2] - D_i^i - s(e^*_2) \right) \right] \right) \right]$$  \hspace{1cm} (49)

The optimal terms for when luck is contractible are:

$$\bar{\kappa}_i^e = 1$$  \hspace{1cm} (50)
Similarly, when only income is contractible these terms are:

\[\bar{K}_i^\gamma = \frac{2a^i\sigma_i^2}{1 + 2a^i\sigma_i^2}\] (51)

The trade-off between insurance and incentives is now expressed as a function of \(\sigma_i^2\).

### 2.2.3 The value of the optimal debt contract

In section 2.1 we wrote down three possible linear contracts; for two of these we’ve derived the optimal contract by solving for the trade-off between insurance and incentives: under the third there was no trade-off to make. Based on these solutions we can now write down the value of the contract as a function of luck.

First, the contract where the parties contracted on income:

\[D_i^Y = B_1^{i^*}(v^{i^*}_2(\bar{K}_i^\gamma), r)(1 + r) + \bar{K}_i^\gamma e^t = 0\] (52)

In section 2.1.1 this contract contained the variables efforts and expected income; the reason for this difference with the section above, is that the pay-off is from an ex post perspective. This is the result of the assumption in section 1.2.2: the exerted effort for any \(\bar{K}_i^\gamma\) is anticipated.

Second, is the contract where the parties can contract on luck:

\[D_i^\varepsilon = B_1^{i^*}(v^{i^*}_2, r)(1 + r) + \bar{K}_i^\varepsilon e^t = 0\] (53)

And finally, the contract where the parties can contract neither on luck nor on income:
The pay-off of any particular contract is thus a function of the demand for loans and the insurance offered. When we want to compare the value of each contract however we’re not going to learn very much when we compare the total pay-offs. The reason for this is that the quantity demanded is not the same for each contract, for the terms of the contract shift the demand curve; the consequence is a change in the quantity demanded and a possible change in rates for an upward sloping supply curve. The comparison between totals for the value of the contract is therefore one of apples and oranges. One possible solution to this is to re-write each of the contracts as divided by the amount promised or:

\[
\frac{\epsilon}{E[\epsilon]} = \frac{D_i}{B_i^*(e_i^*, r)(1 + r)}
\]  

(55)

In this way we can re-scale the contracts and compare them on the pay-off per \( \epsilon \). Graphically the pay-offs as a function of luck are:

Figure 2.2.3.a: The contracts with their different pay-offs as a function of luck (left); the contract where income is contractible with the pay-offs for increasing \( \sigma_{it}^2 \) (the slope moves counterclockwise with \( \sigma_{it}^2 \)) as a function of luck (right).
There are four observations worth to make about these two graphs. First is the absence of a premium for risk; how sensitive the contract is to luck does not increase the expected pay-off per €\(^{35}\). This is not unexpected given our starting assumptions: idiosyncratic risk goes unrewarded and the intermediary is perfectly diversified. A second observation (right graph) is that for low variance, the optimal debt contract is equivalent to a contract where the payment is fixed or,

\[
\lim_{\sigma_i \to 0} \overline{K}_{t}^{y} = \frac{2a_i \sigma_i^2}{1 + 2a_i \sigma_i^2} = 0
\]  

(56)

Third, for a very high variance, the insurance parameter of the optimal debt contract is equivalent to the contract where only luck is insured.

\[
\lim_{\sigma_i \to \infty} \overline{K}_{t}^{y} = \frac{2a_i \sigma_i^2}{1 + 2a_i \sigma_i^2} = 1
\]  

(57)

Finally, the slope of the contractual pay-off is informative as a higher slope implies more redistribution from those who were ‘lucky’ to those who suffered a bout of bad luck\(^{36}\).

2.2.4 The welfare of the debtor and the optimal debt contract

From the role of \(\overline{K}_{t}^{e}, \overline{K}_{t}^{y}(\sigma_i^2)\) and \(\overline{K}_{t}^{0}\) in the contract, it is easy to deduce that if,

\[
\overline{K}_{t}^{e} > \overline{K}_{t}^{y}(\sigma_i^2) \geq \overline{K}_{t}^{0}
\]  

(58)

\(^{35}\) The expected pay-off here is where the line crosses the ordinate.

\(^{36}\) This fact allows us to calculate the premium for the right to buy-back at the opportunity cost in Chapter 3.
Then,

\[ D^x_1 > D^y_1 \geq D^0_1 \] (59)

For the contract that insures only luck cannot affect incentives, whilst in \( D^y_1 \) there is a trade-off to be made; and for very low values of \( \sigma_i^2, \bar{\kappa}_i^y \) and \( \bar{\kappa}_i^0 \) are indistinguishable or \( \bar{\kappa}_i^y (\sigma_i^2) \approx \bar{\kappa}_i^0 \).

This, however, is not the whole story. For as \( \sigma_i^2 \) increases, the effort of the actor diminishes due to \( \bar{\kappa}_i^y (\sigma_i^2) \) and with effort goes income; the marginal cost of insurance can now be defined as:

\[
MC = -\frac{\delta e^*_i (\bar{\kappa}_i^y) - \delta e^*_i (\bar{\kappa}_i^y)^2}{\delta \sigma_i^2}
\] (60)

Graphically, we see that the marginal cost of insurance, rises steeply in the beginning to drop off later. The marginal benefits of insurance are in the beginning likely outweighed by the costs; only when these costs start to decrease, should we expect to see \( D^y_1 > D^0_1 \).

\[ \text{Figure 2.2.4.b: The graphs above show the fraction of income of the debtor that is insured as function of } \sigma_i^2 \text{ and the marginal cost of insurance as a function of variance. The different lines are the result of varying the risk-aversion of the debtor.} \]
Figure 2.2.4.a: The graphs above show the utility-curve of the debtor as a function of $\sigma_i^2$ under the contract where income is insured (solid line) and under the contract where no insurance is offered (dashed). The different lines are the result of varying the parameters; the title of the graph states which parameter was varied\(^{37,38}\).

This conclusion is illustrated when the utility curves under the two contracts are made a function of $\sigma_i^2$; as can be seen above, the conclusion holds for when the parameters of the model are varied: for very low values of $\sigma_i^2$, $D_i^V \approx D_i^0$. When transaction costs can be included in the model we should expect to see $D_i^0 > D_i^V$, for very low values of $\sigma_i^2$.

What this comparison illustrates, is that it depends on $\sigma_i^2$ which contractual terms maximize the welfare of the borrower when a contract contingent on luck is unavailable\(^39\).

\(^{37}\) The graph which depicts the change in utility as a result of a change in risk-aversion is not very informative; for the utility curves of the same contract cross. This result is a consequence of the choice of utility function we made earlier. The graph however is included for the sake of completeness, but will be omitted later.

\(^{38}\) The time preference of the actor is varied from 0.1 to 0.9 in steps of 0.2 or $\beta^i \sim (0.1; 0.9; 0.2)$, similarly: $\alpha^i \sim (0.1; 1; 0.2)$, $\mu_i \sim (1; 0.5)$ and $r \sim (0; 0.01; 0.1; 0.01)$, the default values for $\beta^i, \alpha^i, \mu_i, r$ are for no particular reason: 0.9; 1; 0.1. If nothing else is stated, then these will be the values used for the parameters to graph the curves.
Experience and Equilibrium

There is however another way to approach this problem and this is to look at it from the perspective of what the equilibrium construct does here; for recall that one of our early assumptions was that the consequence of effort is predictable and thus the effort under each contract is anticipated. This assumption however, presumes a certain amount of experience on part of the actors about the world of the model whereas this experience can vary with $\sigma_i^2$.

For when $\sigma_i^2$ is high, the actors live in a world where there is a fairly low signal-to-noise ratio; in their experience, whatever happens, is mostly due to luck and very little is due to effort. In such a world, the actors will not exert much effort as the consequence of their effort is indistinguishable from luck. The contracts in such a world will therefore be mostly about the allocation of risk. On the other hand, when $\sigma_i^2$ is low, there is a very high signal-to-noise ratio and the experience of the actors in such a world is that their effort matters and they will exert more of it. When $\sigma_i^2$ is low the contracts will mostly be about effort rather than the allocation of risk.

The use of the equilibrium construct (see above section 1.1) in the model, therefore substitutes for the experience of the actors and hence we only see $\sigma_i^2$ as the parameter in the optimal contract. As Schumpeter put it:

“We assume all this experience to be nonexistent, and reconstruct it ab ovo, as if the same people, still having the same culture, tastes, technical knowledge and the same initial stocks of consumers’ and producers’ goods, but unaided by experience, had to find their way towards the goal of the greatest possible economic welfare by conscious and rational effort.”

---

39 What applies to the welfare of the borrower, also applies to the welfare of the intermediary; the welfare of the borrower is maximized by shifting the demand for loans out and this maximizes profit (see section 2.2.1).

40 Schumpeter (2012) p.10
From this perspective which contractual terms are optimal also depends on $\sigma_{t}^{2}$, but the story behind it is a different one. Both stories are of course true, but a change in perspective can reveal something that would otherwise remain hidden.

2.2.5 The optimal debt contract and default rules

The question of what the optimal debt contract is part of a broader discussion within the economic interpretation of contracts on what to do in the event of unforeseen contingencies or $\varepsilon_{\mu=0}$ (see also Triantis 1999). The answer provided above is twofold: First, when luck is contractible, then contract on luck. Second when transaction-costs are high, then the optimal terms are those that provide insurance against $\varepsilon_{\mu=0}$ when $\sigma_{t}^{2}$ is high or $\kappa_{t}^{y}$, but otherwise provide none or $\kappa_{t}^{0}$.

The perspectives above provide two complementary explanations for this second answer. The first reason is that the (re-)allocation of risk is not free; when the parties to the contract have (virtually) the same expected utility under each contract\(^{41}\), then allocating or re-allocating risk, is an activity that cannot result in a gain and no resources should be devoted to that purpose. Therefore, when the parties have not provided for the contingency or they’ve written $\kappa_{t}^{0}$ instead of $\kappa_{t}^{y}$, when $\sigma_{t}^{2}$ is low, then the job of the courts should be to enforce the contract as it stands. Or as Holmes’ already argued in the case of civil liability or torts: “sound policy lets losses lie where they fall, except where a special reason can be shown for interference. The most frequent of such reasons is that the party who is charged has been to blame.”\(^{42}\).

The second reason is that when $\sigma_{t}^{2}$ is low, the presumption is that the parties have contracted on effort; to shift the risk away from the party that should exert effort, would be counter to the

\(^{41}\)Note that this is the case before the costs of contracting are included for low $\sigma_{t}^{2}$ (see above).

\(^{42}\)Holmes 2012 p.31
intention of the parties when they wrote the contract and allocated the risk. When seen in this perspective, the contract is complete and should be enforced as it stands.

Similarly, Holmes states that: “experience is the test by which it is decided whether the degree of danger attending given conduct under certain known circumstances is sufficient to throw the risk upon the party pursuing it”\(^{43}\). The logic of Holmes is the same as in that of the canonical tort model. The experience with the circumstances is represented by the expected damage curve as a function of all levels of precaution and it is experience – in the form of the marginal Hand Rule – which tells us what the level of precaution should be (see also Schumpeter above)\(^{44}\). In the unilateral model it is then a question of either imposing strict liability on the party that can take precaution or to impose a duty of negligence at the efficient level of precaution (see Shavell 2004 p.178-181); both will lead to the efficient outcome. Similarly, whether the parties here make the party exerting effort strictly liable or impose only liability for \(\varepsilon_{\mu=0}\) when effort is below a certain threshold, is irrelevant from the perspective of total welfare. The contract is complete and should be enforced as it stands.

**Frustration of purpose**

We can illustrate the use of this rule, by examining the doctrine of frustration of purpose; in particular when it comes to the question when it should apply and when it should not: the question is identical to when to substitute \(K_t^{y'}\) for \(K_t^0\). The textbook case here is the case of *Krell v. Henry*\(^{45}\) (see Cooter & Ulen 2012 p.353); the case of *Krell v. Henry* is part of the so-called ‘coronation-cases’, ‘so-called’, because these involved contracts where due to the postponement of the coronation, due to illness of the King, performance became pointless or so it was argued. In

---

\(^{43}\) Holmes (2012) p.90

\(^{44}\) In terms of the tort model therefore also: \(\frac{\delta p(x)A}{\sigma^2} < 0\)

\(^{45}\) In *Krell v. Henry* [1903] 2 KB 740 three conditions are given by Vaughan Williams LJ; first, the foundation of the contract has to be determined; second it has to be determined that the contract was prevented; and third the event has to have been outside the contemplation of the parties.
this particular case a room was rented with the implicit purpose to watch the parade in exchange for a substantial sum.

The question that came before the court was whether the room or the parade was the foundation of the contract; if the latter, the contract was frustrated, if the former it could continue. The comparison made here was between that of contract for hire of a cab to drive to Epsom on Derby Day and the contract to rent the room overlooking the parade during the coronation.

The conclusion of this comparison was that there was a crucial difference between the cab and the room; for the cab could have been any cab and the drive to Epsom could have been for any purpose, whilst the use of the room and the location of the room made it far more qualified. Moreover, “under the cab contract, the hirer, even if the race went off, could have said, "Drive me to Epsom; I will pay you the agreed sum; you have nothing to do with the purpose for which I hired the cab,” and that if the cabman refused he would have been guilty of a breach of contract, there being nothing to qualify his promise to drive the hirer to Epsom on a particular day.”

In contrast, “in the case of the coronation, (...) it is the coronation procession and the relative position of the rooms which is the basis of the contract as much for the lessor as the hirer”

In terms of the above, the former is also a question of effort, whilst in the latter whether or not the coronation occurs is a question of sheer luck. In the former therefore, there is little reason to substitute $\bar{r}_t^Y$ for $\bar{r}_t^0$, whilst in the case of the latter, $\bar{r}_t^Y$ does maximize total welfare ex ante.

Support for this reading can be found throughout the case law. For example in the original case of Taylor v. Caldwell we find the application of $\bar{r}_t^Y$ when it is a question of sheer luck:

---

46 Krell v. Henry per Vaughan Williams LJ
47 ibid
48 This same decision with the very same reasoning down to the analogy and the judges can be found in the case of Herne Bay Steam Boat v Hutton [1903] 2 KB 683; the defendant had hired a steamboat to take passengers to on a cruise through the harbor to watch the naval review; when the review was cancelled with the coronation, the contract was not frustrated as the cruise could still take place.
49 Taylor v Caldwell (1863) 3 B&S 826
"The principles seems to be that in contracts in which the performance depends on the continued existence of a given person or thing, a condition is implied that the impossibility of performance arising from the perishing of the person or thing shall excuse the performance"

Other cases can also be read in the same vein\textsuperscript{50}\textsuperscript{51}; when the presumption is that the parties have contracted on effort, rather than sheer luck, there can be no frustration of purpose\textsuperscript{52}.

2.3 The optimal debt contract when information is produced by the intermediary

Above the implicit assumption was that information is free and fixed; this will be relaxed here. In section 1.2.1 we made a distinction between type risk or $\epsilon_\theta$ and individual risk or $\epsilon_i$. From the perspective of an outsider observing the model there is no distinction between these two, from the perspective of the actors in the model, however, there is: when observed from an aggregate perspective we call it $\epsilon_\theta$ and when observed from an individual perspective we call it $\epsilon_i$. The two perspectives are equivalent to the perspective of the intermediary and the perspective of the individual debtors; the intermediary can observe all loans, whilst the debtor observes but a few at most. In other words: from the perspective of the debtor, the mean of $\epsilon_i$ or $\mu_i$ is a random variable\textsuperscript{53}. From the perspective of the debtor therefore:

\textsuperscript{50} Tsakiroglou & Co Ltd v Noble Thorl GmbH [1962] AC 93, HL; Blockage of the Suez canal was no reason to for frustration of purpose, for an alternative route was still available.

\textsuperscript{51} Blackburn Bobbin v TW Allen [1918] 2 KB 467; Import of a good from another country was prevented due to a blockade there: no frustration of purpose, the goods could have been obtained elsewhere.

\textsuperscript{52} See also Triantis (1999) p.109 where a similar observation is made; see however p.102 for the difference with the view expressed here. "the excuses of frustration, (...) referring to ‘the occurrence of an event [or contingency] (...) which seems to encompass those risks to which the parties attached a probability of near zero”.

\textsuperscript{53} The assumption here is that the variance is known; the goal here is not be fully realistic, but to highlight a difference in knowledge as a result of a difference in position see also Hayek (1945).
\[ \mu_i \sim N \left[ \mu_{\theta_i}, \frac{\sigma_i^2}{\nu_i} \right] \]  

(61)

Where \( \nu_i \) are the number of observations the debtor has made. Similarly, from the perspective of the intermediary, the variance depends on the number of observations the intermediary makes:

\[ \mu_i \sim N \left[ \mu_{\theta_i}, \frac{\sigma_\theta^2}{\nu_\theta} \right] \]  

(62)

Where \( \nu_\theta \), is the number of debtors the intermediary has counted; the intermediary can collect the data, but, because the intermediary is well-diversified, does not care to do so for itself. We'll assume here that the cost of counting is subject to declining costs due to economies of scale and that the cost-curve can be described by \( c(\nu_\theta) = \sqrt[\nu_\theta} \) (see also Mishkin 2004 p. 174; Mishkin gives the example of large capital expenditures such as computers; similarly, once you have setup a datacenter, the cost of making an additional entry is trivial). Finally, the intermediary wants to communicate this information to the debtor; the debtor, it is assumed trusts the intermediary in equilibrium. The logic underpinning this assumption is the logic of tit-for-tat (see also Cooter & Ulen 2012 pp. 299-304). If the debtor would not trust the intermediary there would be no collection nor communication of information, this is inefficient, for both parties would gain if the debtor would start out by trusting the intermediary and the intermediary would not violate that trust; if therefore the intermediary and debtor interact together without a clear end in sight, then future interactions will discipline the intermediary not to betray that trust.

In reality, the legal relationship between an intermediary, its debtors, and its creditors, can be a bit more complicated than the assumptions make it out to believe. The intermediary does not
always act in the interest of either nor has to and trust placed in it can be violated. In the case of depositors as creditors, for example, the relationship is made out to be one of ordinary debtor and creditor\textsuperscript{54} and the bank cannot be said to be an agent to act on behalf of a depositor. Nor is it necessarily the case that there is a relationship of trust and confidence between the borrower and the bank, where the bank acts as an agent for the borrower\textsuperscript{55}. It can however be proved that the relationship is one such where undue influence is presumed as was the case in \textit{Lloyds Bank v Bundy}\textsuperscript{56}. And the bank does have a certain responsibility to disclose the full extent of the liability whenever a wife, for example, secures the debts of her husband and makes sure that the agreement to do so was obtained properly\textsuperscript{57,58}.

Nevertheless, the setup of the model here is one where it is in the interest of the intermediary to collect and disseminate the information to the debtor as part of its pursuit for profit\textsuperscript{59}.

\textbf{2.3.1 A mathematical model of information production}

Mathematically we can formalize this by recognizing, that instead of:

\begin{equation}
\exp[-a^i\epsilon_{\mu=0}] = \exp[-a^i\mu_{\mu=0} + \frac{a^i_2}{2}\sigma_i^2]
\end{equation}

We now have:

\textsuperscript{54} \textit{Foley v. Hill} (1848) 2 HLC 28  
\textsuperscript{55} \textit{Natwest Bank v Morgan} [1985] AC 686 HL  
\textsuperscript{56} \textit{Lloyds Bank v Bundy} [1975] QB 326  
\textsuperscript{57} \textit{RBS Plc v Etridge (No 2)} [2002] 2 AC 773  
\textsuperscript{58} \textit{Barclays Bank v O Brien} [1994] 1 AC 180 HL  
\textsuperscript{59} There is a setup where the information collected does not make the borrower worse-off, whilst it does offer the possibility of making the intermediary better-off; the logic is that the same information can be used to induce the borrower to borrow more than optimal, because the borrower is told to be ‘lucky’. In other words, the bank will collect information to lower $\sigma_i^2$ so as to be able to overstate $\mu_i$. 

\[
\exp[-a^i(e^i_{\mu=0} + \mu_i)] = \exp[-a^i \mu_\theta + \frac{a^i}{2} \frac{\sigma_i^2}{\sigma_\theta^2} \left(1 + \frac{1}{\sqrt{\nu_i}}\right)]
\] (64)

And when the debtor trusts the intermediary, this becomes \(^{60}\):

\[
\exp[-a^i(e^i_{\mu=0} + \mu_i)] = \exp[-a^i \mu_\theta + \frac{a^i}{2} \sigma_\theta^2 \left(1 + \frac{1}{\sqrt{\nu_\theta}}\right)]
\] (65)

It is therefore as if the variance of the debtor has been multiplied by \(1 + \frac{1}{\sqrt{\nu_\theta}}\). Using this information it becomes possible to derive a new demand for loans similar to the one above:

\[
B^i_1 = \frac{a^i \left(e^i_2 - e^i_2^{*2} + \mu_i - \frac{1}{2} a^i \sigma_i^2 \left(1 + \frac{1}{\sqrt{\nu_\theta}}\right) (\bar{\kappa}_i - 1)^2\right) + \log \left[\frac{1 + \rho^i}{1 + r}\right]}{a^i (2 + r)}
\] (66)

As we will want to compare the equilibrium with production of information to the one without, we will substitute \(\nu_\theta\) with \(\nu_\theta + 1\) in the demand and in the cost function; the interpretation is that one observation is already present: namely the debtor’s observation. This allows us to substitute \(\sigma_i^2\) with \(\sigma_\theta^2\) for \(1 + \frac{1}{\sqrt{\nu_\theta+1}} \leq 2\). Perceived variance is now equivalent to variance in the model without information production for the same numerical values of \(\sigma_i^2\) prior to production; to keep the notation consistent: \(\sigma_i^2 \equiv \sigma_\theta^2\). Therefore:

\[
B^i_1 = \frac{a^i \left(e^i_2 - e^i_2^{*2} + \mu_i - \frac{1}{4} a^i \sigma_\theta^2 \left(1 + \frac{1}{\sqrt{\nu_\theta+1}}\right) (\bar{\kappa}_i - 1)^2\right) + \log \left[\frac{1 + \rho^i}{1 + r}\right]}{a^i (2 + r)}
\] (67)

\(^{60}\) \(\sigma_i^2 \equiv \sigma_\theta^2\)
The multiplication of the cost by \( \frac{1}{n} \) here simply means that the cost of counting is shared among the \( n \) debtors the intermediary has; this is equal to the number of debtors in the model (see section 1.2.1). The problem of the intermediary can now be re-written as:

\[
\max_{v_{\theta}} b^{1*}_{\theta}(v_{\theta})(r - r_c) - \frac{1}{n}\sqrt{v_{\theta} + 1}
\]

(70)

And we’re now ready to solve for the optimal \( \bar{\kappa}_t \) in the same way as above; the optimal effort has not changed, therefore:

\[
\bar{\kappa}_t^\xi = 1
\]

(71)

\[
\bar{\kappa}_t^\gamma = \frac{a^i \sigma^2(1 + \sqrt{v_{\theta} + 1})}{\sqrt{v_{\theta} + 1 + a^i \sigma^2(1 + \sqrt{v_{\theta} + 1})}}
\]

(72)

\[
\bar{\kappa}_t^0 = 0
\]

(73)

The optimal contract terms are identical, regardless whether one chooses to solve for the intermediary or the borrower (see section 2.2.1). Further, note here that provided \( a^i > 0 \) and
$\sigma_t^2 > 0$, the slope of the contract under information production is flatter than when no information is produced.

\[
\frac{a^i \sigma_{\theta}^2 (1 + \sqrt{v_{\theta} + 1})}{\sqrt{v_{\theta} + 1} + a^i \sigma_{\theta}^2 (1 + \sqrt{v_{\theta} + 1})} < \frac{2a^i \sigma_t^2}{1 + 2a^i \sigma_t^2}
\]  (74)

Only at the limit are the two alike and is it similar to the other two contracts as above:

\[
\lim_{\sigma_t^2 \to \infty} \overline{K}_t^\gamma = \frac{a^i \sigma_{\theta}^2 (1 + \sqrt{v_{\theta} + 1})}{\sqrt{v_{\theta} + 1} + a^i \sigma_{\theta}^2 (1 + \sqrt{v_{\theta} + 1})} = 1
\]  (75)

\[
\lim_{\sigma_t^2 \to 0} \overline{K}_t^\gamma = \frac{a^i \sigma_{\theta}^2 (1 + \sqrt{v_{\theta} + 1})}{\sqrt{v_{\theta} + 1} + a^i \sigma_{\theta}^2 (1 + \sqrt{v_{\theta} + 1})} = 0
\]  (76)

The explanation for the difference is, that the production of information acts as a substitute for insurance as can be glanced from $\frac{\delta K_t^\gamma(v_{\theta})}{\delta v_{\theta}} < 0$, provided: $v_{\theta} > -1, a^i > 0$ and of course $\sigma_t^2 \neq 0$.

Or to put it more plainly: reassurance substitutes for insurance.

2.3.2 Information production by the intermediary

Where the intermediary can be described as choosing $v_{\theta}^\Pi^*$ as equating the marginal benefit and cost of the collection of information:

\[
\frac{\delta B_t^i (v_{\theta}^\Pi^*, \overline{K}_t)}{\delta v_{\theta}} (r - r_c) = \frac{1}{n} \frac{1}{2 \sqrt{v_{\theta}^\Pi^* + 1}}
\]  (77)
The marginal benefit is here equivalent to the marginal value of an increase in volume. And when solved for the three contracts \( v^\pi_{\theta} \), the information gathered by the intermediary can be expressed as a function of the spread, the number of customers, their risk-aversion and the variance of their income.

\[
v^\pi_{\theta} = 0
\]  

\[
v^\pi_{\theta} = \frac{1}{4}a^i\sigma_\theta^2 n (r - r_c) - (1 + 2a^i\sigma_\theta^2)(2 + r)\
\frac{(2 + r)(1 + a\sigma_\theta)^2}{\sqrt{(2 + r)(1 + a^i\sigma_\theta)^4}}
\]  

\[
v^\pi_{0^+} = \frac{1}{4}a^i\sigma_\theta^2 n (r - r_c) - (2 + r)
\frac{(2 + r)}{(2 + r)}
\]  

Note that the information collected is information in aggregate; it is information about all the debtors without that the intermediary necessarily knows the type of the debtor. The intermediary can therefore know more about the debtor in one dimension, because it knows more about all the debtors as a group (see for the explanation section 1.2.1).

And because information is a substitute for insurance, insurance is also a substitute for information; the amount of information collected in equilibrium under the contract without insurance is higher than the amount of information collected under the contract with,

\[
v^\pi_{0^+} > v^\pi_{\theta}^y
\]

provided, that the number of debtors is sufficiently large. The reason for this is purely mathematical; the cost function has an implicit coefficient of 1 and the income of the debtor
cannot exceed \( \frac{1}{4} \); for low values of \( n \) the cost is therefore prohibitive. For the economic assumptions to be satisfied and the intermediary to produce information \( n \) has to be large. Graphically, we can see the marginal cost curve dropping considerably to more plausible values as \( n \) is increased:

![Marginal Cost of Information](image)

*Figure 2.3.2.a: The figure depicts different marginal cost-curves for different values of \( n \); the values range from \( n = 10 \) to \( n = 20 \) in steps of 20: the lower \( n \) is, the higher the marginal cost of information.*

### 2.3.3 The optimal production of information

The production of information by the intermediary, however, is not necessarily equivalent to the optimal production of information. For the marginal benefits to the intermediary are lower than the marginal benefits to the borrower. We can derive the optimal amount of information, by re-writing the budget constraint of the borrower so that the borrower bears the cost of information production and then solve for \( \nu_\theta \); the terms of the contract where \( \nu_\theta \) is fixed or c.p. remain the same as these specify the trade-off between insurance and incentives and these marginal benefits and costs are unaffected by this modification of the problem.

\[ c_2^i = E[y_2^i] - D_1^i - \frac{1}{n}\sqrt{\nu_\theta} + I(1 + r); \] the cost is incurred in the first period, but has to be repaid in the second period, therefore it is multiplied by the time value.

\[ \nu_\theta \] From a mathematical point of view \( \bar{k}_1 \) is the solution to the \( FOC_{\bar{k}_1} \) and \( \nu_\theta \) to \( FOC_{\nu_\theta} \); the sole difference is in the final slope of \( \bar{k}_1 \) as a function of the other parameters when \( \nu_\theta \) is eliminated.
\[ v_{\theta}^{0*} = \frac{\frac{1}{4} a^i \sigma_{\theta}^2 n - (1 + r)}{(1 + r)} \]  

\[ v_{\theta}^{\pi*} = \frac{a^i \sigma_{\theta}^2 \left( \frac{1}{4} - \left( 1 + a^i \sigma_{\theta}^2 \right) \frac{a^i \sigma_{\theta}^2 (1 + r)}{n(1 + a^i \sigma_{\theta}^2)^2} \right) - (1 + r)(1 + 2a^i \sigma_{\theta}^2)}{(1 + r)(1 + a^i \sigma_{\theta}^2)^2} \]  

In the first case, it is clear that there is less information produced than is strictly optimal from the perspective of the consumer; in the second case the limit to \( n \) of the difference, tells that as \( n \) increases, \( v_{\theta}^{\pi*} - v_{\theta}^{0*} \) also increases. Both are premised on the assumption that \( r - r_c \) is not too large.

Based on this assumption we can also deduce that:

\[ \bar{K}_t^y \left( v_{\theta}^{0*} \right) < \bar{K}_t^y \left( v_{\theta}^{\pi*} \right) \]  

For recall from section 2.3.1: \( \frac{\delta \bar{K}_t^y (v_{\theta})}{\delta v_{\theta}} < 0 \). Both parties consider information and insurance to be substitutes, however, the marginal cost of insurance declines faster for the intermediary than it does for the borrower. The reason here is that the borrower splits income between the first and the second period, whilst the intermediary only cares about the income used for consumption in the first period multiplied by the spread. Consequently, the intermediary will invest less in information and more in insurance.

2.3.3 A brief note on the welfare properties under information production
Now that the model has been solved it is possible to, briefly, touch upon how the two optimal debt contracts discussed earlier affect the welfare of the borrower and the profits of the intermediary; the profits are included now the allocation of resources plays a role.

---

As above, the contract for luck is still preferable to either.

Two new parameters have been added: the “cost of funds” or $r_c \sim (0; 0.09; 0.01)$ and $n \sim (10.000; 100.000; 200.000)$ or “population”. The default values for $\beta^l, \alpha^l, \mu, r, r_c, n$ are: 0.9; 1; 0.1; 0; 10.000.
Figure 2.3.3.a: The figure depicts utility (left) and profit (right) as a function of variance, where the dashed line is for the fixed contract and the solid line is for the contract with insurance of income.

What we can in figure 2.3.3.a above, is that there is a difference with the results of the model in the previous section. The main difference is that the two contracts seem much closer than before and there are two reasons for this effect. First, overall, the variance has been reduced when compared to the previous section; the numbers on the abscissa are the same: these numbers refer to the perceived variance prior to the production of information. This first effect means that the slope of $D_1^Y$ or $\overline{\kappa}_t^Y$ is now flatter than before. Second, is that information is a substitute for insurance, and as we've seen above, this means that more is produced under $D_1^{Y0}$ than under $D_1^Y$.

The conclusions made on the basis of the previous model now are also slightly modified, for the choice of default rule or $\overline{\kappa}_t^Y$ vs. $\kappa_t^0$, now also depends on whether or not the production of
information is regulated. Recall, that there is an underproduction of information from the perspective of the borrower.

Finally, with regard to the profits of the intermediary, we can make the observation that profits are overall positive and this indicates that the production of information is a profitable activity for the intermediary. Within the model, the decision to allocate resources to the production of information is therefore a good one and is to the benefit of both parties.

The conclusion here is that the production of information is desirable, because it substitutes for insurance.

---

65 The profit at some points is negative, but this has more to do with the choice of values for the parameters than with anything else; the same holds for the crossing of the profit-curves under \( D_t^0 \). The increase in the cost of funds decreases the profit-margin and this can have some odd effects. These ‘odd effects’ here are primarily due to the relative difference between the profit-margin and the cost of information.
Chapter 3: The right to buy-back premature debt

“No man’s credit is as good as his money.”

E.W. Howe

3.1. The right to buy back premature debt

There are three equations that define the optimal debt contract. The first equation tells us that it consists of flexible and a fixed part:

\[
E[D_1^{i*}] = f + \kappa_i E[y^{i*}_2 (x^i)]
\] (85)

The second equation tells us that the expected value of the contract is equal to the promise to repay.

\[
E[D_1^{i*}] = B_1^{i*} (1 + r)
\] (86)

Finally, the third equation tells us how much is actually repaid at the date of repayment:

\[
D_1^i = B_1^{i*} (e_2^{i*}, r) (1 + r) + \kappa_i e_{\mu=0}^i
\] (87)

There are therefore two ways in which a right to re-purchase at the opportunity cost to the creditor can be exercised. It can be either at \(E[D_1^{i*}]\) or at \(D_1^i\); the difference between these two is, whether or not \(e_{\mu=0}^i\) has occurred, and who has observed and who has not at the time of the exercise.
When the intermediary observes \( \varepsilon_{\mu=0}^i \) before the debtor does, the opportunity cost is \( D_1^i \) and nothing extraordinary can happen\(^{66}\). Similarly, when both observe \( \varepsilon_{\mu=0}^i \) at the same time; what has been promised to the intermediary will be repaid. However, when the debtor observes \( \varepsilon_{\mu=0}^i \) before the intermediary does, then the opportunity cost is \( E[D_1^{i*}] \) and the opportunity cost of the debtor is \( D_1^i \). When \( E[D_1^{i*}] > D_1^i \), the debtor \( E[D_1^{i*}] < D_1^i \), and the debtor will re-purchase. This is the case we will discuss now by first defining the price of the right to re-purchase. There is one caveat here: namely this section assumes that there are no income-effects from improved incentives\(^{67}\).

### 3.1.1 The price of the right to re-purchase

We can define the expected value of the contract from the perspective of the intermediary as the integral of the pay-off for all \( \varepsilon_{\mu=0}^i \), multiplied by the probability of occurrence.

\[
E[D_1^{i*}] = B_1^{i*} (e_2^{i*}, r)(1 + r) + \int_{-\infty}^{\infty} k_i \varepsilon_{\mu=0}^i f_n(\varepsilon_{\mu=0}^i) d\varepsilon_{\mu=0}^i
\]

(88)

We can thus omit the fixed part, and re-write this as:

\[
E[\varepsilon_{\mu=0}^i] = \int_{-\infty}^{0} k_i \varepsilon_{\mu=0}^i f_n(\varepsilon_{\mu=0}^i) d\varepsilon_{\mu=0}^i + \int_{0}^{\infty} k_i \varepsilon_{\mu=0}^i f_n(\varepsilon_{\mu=0}^i) d\varepsilon_{\mu=0}^i
\]

(89)

\(^{66}\) There is room for opportunism if the intermediary is allowed to lie in the same vein as in section 2.3 above: see fn 59.

\(^{67}\) When the debtor can re-purchase the debt upon observing \( \varepsilon_{\mu=0}^i \), the opportunity cost of not doing so also includes the improved incentives for effort. This assumption therefore results in the debtor re-purchasing more often than that the debtor actually would and as a consequence paying a smaller premium to do so. From an ex ante perspective the effect seems very small and consequently also very difficult to calculate. See also though Schumpeter (2012) in section 2.2.4.
And when the right to re-purchase can be exercised at $E[D^i_t]$ this becomes:

$$E[\epsilon_{i=0}] = \int_{-\infty}^{0} \bar{K}_i \epsilon_{\mu=0} f_n(\epsilon_{\mu=0}) d\epsilon_{\mu=0}$$  \hspace{1cm} (90)

The result is that:

$$E[D^i_t] > B^i_2(e^i_2, r)(1 + r) + \int_{-\infty}^{0} \bar{K}_i \epsilon_{\mu=0} f_n(\epsilon_{\mu=0}) d\epsilon_{\mu=0}$$  \hspace{1cm} (91)

The solution to this problem is to add back the difference as a fixed part of the contract (see section 1.3); it is this difference that is the price of the right to re-purchase for $i$ or $R_i$.

$$R_i = \int_{0}^{\infty} \bar{K}_i \epsilon_{\mu=0} f_n(\epsilon_{\mu=0}) d\epsilon_{\mu=0}$$  \hspace{1cm} (92)

When we solve for the integral,

$$R_i(\bar{K}_i, \sigma_i) = \frac{\bar{K}_i \sigma_i}{\sqrt{2\pi}}$$  \hspace{1cm} (93)

we find that the price or premium is proportional to the insurance and the root of the variance or the standard deviation. Intuitively, this is plausible, for the premium tells us that the more the contract is about redistribution between borrowers (see section 2.2.3), the higher the premium; similarly, the more the contract is about insurance rather than effort, the higher the premium (see section 2.2.4).
3.1.2 The optimal debt contract with the right to re-purchase

We are now ready to re-derive the optimal debt contract with the right to re-purchase included. First, we re-write the problem of the borrower:

$$
\max_{c_i'} Eu \left[-\exp\left[-a^i c_i'\right] + \beta_i^i \left(-\exp\left[-a^i \left(E[y_i^i'] - D_i - R_i - s(e_i')\right]\right)\right]\right] \quad (94)
$$

Then we solve for the demand for loans:

$$
B_1^{iR^*} = \frac{2\sqrt{\pi} \left(a^i \left(e_2^i - e_1^{i+2}\right) + \mu_i - \frac{1}{2} a^i \sigma_i^2 (\kappa_i - 1)^2\right) + \log \left[\frac{1 + \rho_i}{1 + r}\right]}{2a^i \sqrt{\pi} (2 + r)} - a^i \kappa_i \sigma_i \sqrt{2} \quad (95)
$$

When we contrast this with the demand for loans in section 2.2.2, we find the demand for loans with a premium, to be lower than the demand for loans without c.p\(^{68}\):

$$
B_1^{i*} = \frac{a^i \left(e_2^i - e_1^{i+2}\right) + \mu_i - \frac{1}{2} a^i \sigma_i^2 (\kappa_i - 1)^2\right) + \log \left[\frac{1 + \rho_i}{1 + r}\right]}{a^i (2 + r)} \quad (96)
$$

The difference between these two can be ascribed to the premium for:

$$
B_1^{iR^*} - B_1^{i*} = \frac{\kappa_i \sigma_i}{\sqrt{2\pi} (2 + r)} \quad (97)
$$

Similarly, we can now solve for the optimal terms of the contract for both the intermediary and the borrower (see section 2.2.2):

\(^{68}\) See fn 67
Where we find that under the contract with the right to buy back, usually, less insurance is offered than under the contract with:

\[ \overline{K}_t^{R^E} \geq \overline{K}_t^{R^E} \]  \hspace{1cm} (101)

\[ \overline{K}_t^{R^Y} \geq \overline{K}_t^{R^Y} \]  \hspace{1cm} (102)

In case of the contract without insurance there is no difference:

\[ \overline{K}_t^0 = \overline{K}_t^{R^0} \]  \hspace{1cm} (103)

### 3.1.3 A brief welfare comparison

From the preceding section we know that premium decreases the amount of insurance; consequently we expect to find that the gap between the contract with insurance for income and the contract without any has decreased. This difference decreases further with variance for the premium increases with variance. We can see this result in figure 3.1.3.a, where the outcome
Figure 3.1.3.a: The contracts (right) with income insurance (solid) and without any insurance (dashed), compared to the same contracts with the right to buy-back the debt at the opportunity cost to the intermediary (left).
here is compared to the outcome in the second chapter.

The conclusion of this comparison is that the effect of the right to buy back is a negative one, as it decreases the amount of insurance; the reason can be found in that the price of the premium is contingent on the amount of insurance.

3.1.4 The right to buy back debt and breach of contract

The discussion whether or not a debtor should be allowed to re-purchase the debt at the opportunity cost to the creditor, is paralleled by the discussion of whether or not parties should be allowed to breach and pay damages\footnote{The usually recommended measure for damages in case of breach, are perfect expectation damages. In the case of the optimal debt contract, the perfect expectation damages are ex ante equivalent to the opportunity cost of the creditor (see Cooter & Ulen 2012 p.309). Ex post however, the intermediary would have preferred performance, and as this option is not available, a premium has to be added.} or should be forced to perform the contract instead.

What this chapter illustrates, is, that there is interdependence between the terms of the contract or default rules and the production of information on the one hand, and the option (see also Mahoney 1995) to re-purchase or the right to breach on the other. Graphically, we can illustrate this effect on the terms, by subtracting the terms here from the terms of the model in section 2.2:

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.1.3.b.png}
\caption{The distance between the contractual terms here and in section 2.2 illustrated as a function of variance.}
\end{figure}
What we find now is that the effect is non-linear. For $D_1^Y$, the problem first gets worse for low values of $\sigma_i^2$, before the effect on the terms starts to decrease; for $D_1^\sigma$, the effect on the terms decreases gradually. It is superfluous to note that for $D_1^0$ nothing changes.

We’re now ready to draw everything together and conclude the thesis by answering the question asked at the beginning:

“What, if any, are the economic justifications for (not) giving debtors the right to buy back pre-mature debt at the opportunity cost to the creditor?”

3.2 Conclusion

At the start of this thesis we noted that the debtor and creditor are symmetrical in the contract. At the end of the thesis we find that there is no objection to the right to re-purchase the debt at the opportunity cost to the creditor, provided that this symmetry is not violated. If that is the case, the terms of the contract and the production of information are independent from the remedy for breach; in the case of the debt contract the payments in aggregate sum up to the payment that was expected by the borrower and the intermediary.

When this symmetry is violated, as is the case in this chapter, we find a tension to exist between the terms of the contract and the chosen remedy, where the role of information production plays merely a subsidiary role. For from the second chapter we’ve learned that information is merely a substitute for insurance. Now that the price of insurance goes up, more information is produced and more information should be produced as without the right incentives there is an underproduction of information. The increase in the production of information however is not a welfare gain; there is still a net loss for the reason, for the increase is due to a change in relative prices. Were the price of information to drop, then there would be a net gain.
The tension that now exists is analogous to the tension between damages for breach and frustration of the purpose of the contract. For the substitution of damages is least costly when the contract is on effort and $\sigma_t^2$ is low, whilst for doctrines such as frustration should only be allowed when $\sigma_t^2$ is high. For contracts on luck we would therefore like to see specific performance, whilst for contracts on effort damages are not detrimental. Ideally therefore, for everyday contracts, the right to re-purchase could be included, whilst for those more adventurous contracts, specific performance should be the remedy of choice\(^7\). This, however, is not the sole margin along which this decision is to be made (see for example Mahoney 1995 & 2000): high transaction costs are another.

In conclusion, the answer should ideally be no, but in practice this boils down to a qualified yes.

\(^7\) See Gilson (2003); in a way for the high variance contracts we already see such arrangements; through the taking of security and various other instruments, specific performance can generally be assured.
Bibliography

Articles


Books


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Shavell S, Chapter 8 of “Foundations of Economic Analysis of Law”, Harvard University Press 2004


Other

Ichino, A, Slides Labour Law & Economics,

ANNEX: Programming code
A new mathematical model of information production in chapter 2

\[
\text{UtilityConsumption}[c1_] := \\
\quad -\text{Exp}[-a*(c1)] + B*(-\text{Exp}[-a*(e2 - e2^2 + mu - c1*(1 + r))]*\text{Exp}[(a*(1 - k))^2 + ((\sigma^2)/4)*(1 + (1/(\text{Sqrt}[v + 1])))])
\]

Optimal Consumption

\[
\text{Reduce}[[\text{D}[\text{UtilityConsumption}[c1], c1] == 0, v > 0, a > 0, \sigma > 0, B > 0, B < 1], c1, \text{Reals}]
\]

\[
a > 0 \land 0 < B < 1 \land r > -1 \land \sigma > 0 \land v > 0 \land
\]

\[
c1 = \frac{1}{2 a + \sigma r} \left( a e2 - a e2^2 + a mu - \frac{a^2 \sigma^2}{4} + \frac{1}{2} a^2 k \sigma^2 - \frac{1 - a^2 k^2 \sigma^2}{4 \sqrt{1 + v}} + \frac{a^2 k^2 \sigma^2}{2 \sqrt{1 + v}} - \frac{a^2 \sigma^2}{4 \sqrt{1 + v}} + \log \left[ \frac{1}{B (1 + r)} \right] \right)
\]

\[
\text{DemandConsumption}[e2_, k_, v_] := \\
\quad \frac{1}{2 a + \sigma r} \left( a e2 - a e2^2 + a mu - \frac{a^2 \sigma^2}{4} + \frac{1}{2} a^2 k \sigma^2 - \frac{1 - a^2 k^2 \sigma^2}{4 \sqrt{1 + v}} + \frac{a^2 k^2 \sigma^2}{2 \sqrt{1 + v}} - \frac{a^2 \sigma^2}{4 \sqrt{1 + v}} + \log \left[ \frac{1}{B (1 + r)} \right] \right)
\]

Utility with equilibrium consumption

\[
\text{Utility}[e2_, k_, v_] := \\
\quad -\text{Exp}\left[-a*\left( a gap - a e2^2 + a mu - \frac{a^2 \sigma^2}{4} + \frac{1}{2} a^2 k \sigma^2 - \frac{1 - a^2 k^2 \sigma^2}{4 \sqrt{1 + v}} + \frac{a^2 k^2 \sigma^2}{2 \sqrt{1 + v}} - \frac{a^2 \sigma^2}{4 \sqrt{1 + v}} + \log \left[ \frac{1}{B (1 + r)} \right] \right)\right] + \\
\quad B*\left(-\text{Exp}\left[-a*\left( e2 - e2^2 + mu - \frac{1}{2 a + \sigma r} \left( a e2 - a e2^2 + a mu - \frac{a^2 \sigma^2}{4} + \frac{1}{2} a^2 k \sigma^2 - \frac{1 - a^2 k^2 \sigma^2}{4 \sqrt{1 + v}} + \frac{a^2 k^2 \sigma^2}{2 \sqrt{1 + v}} - \frac{a^2 \sigma^2}{4 \sqrt{1 + v}} + \log \left[ \frac{1}{B (1 + r)} \right] \right)\right)\right] \times (1 + r)\]

\[
\text{Exp}[(a*(1-k))^2 + ((\sigma^2)/4)*(1 + (1/(\text{Sqrt}[v + 1])))]
\]

Optimal \( k \)

\[
\text{FOCLuck} := \text{D}[\text{Utility}[1/2, k, v], k] \\
\text{FOCIncome} := \text{D}[\text{Utility}[1/2 - k/2, k, v], k] \\
\text{FOCZero} := \text{D}[\text{Utility}[1/2, 0, v], k]
\]
Reduce[{FOCLuck == 0, k, Reals}]
Reduce[{FOCIncome == 0, B > 0, B < 1, sigma > 0, a > 0), k, Reals}
Reduce[{FOCZero == 0), k, Reals]

\[(a = 0 \&\& ((B < 0 \&\& (r < -2 || -2 < r < -1)) || (B > 0 \&\& r > -1)) \&\& v > -1) ||
((B < 0 \&\& (r < -2 || -2 < r < -1)) || (B > 0 \&\& r > -1)) \&\&
((v > -1 \&\& k = 1) || (sigma = 0 \&\& v > -1))\]

\[a > 0 \&\& 0 < B < 1 \&\& r > -1 \&\& sigma > 0 \&\& v > -1 \&\& k = \frac{a \sigma^2 + a \sigma^2 \sqrt{1 + v}}{a \sigma^2 + \sqrt{1 + v} + a \sigma^2 \sqrt{1 + v}}\]

True

FOCLuckBank := D[DemandConsumption[1/2, k, v] \(*(r - rc) - Sqrt[v+1]/n, k]]
FOCIncomeBank := D[DemandConsumption[1/2 - k/2, k, v] \(*(r - rc) - Sqrt[v+1]/n, k]]
FOCZeroBank := D[DemandConsumption[1/2, k, v] \(*(r - rc) - Sqrt[v+1]/n, k]]
Reduce[{FOCIluckBank == 0}, k, Reals]
Reduce[{FOCIncomeBank == 0, k, \[Sigma] > 0, a > 0}, k, Reals]
Reduce[{FOCZeroBank == 0, k == 0}, k, Reals]

\[
\begin{align*}
\text{ProfitLuck}[v_\text{-}] & := \text{DemandConsumption}[1/2, 1, v] \cdot (r - rc) - (1/n) \cdot Sqrt[v+1] \\
\text{ProfitIncome}[v_\text{-}] & := \text{DemandConsumption}[1/2 - \left(\frac{\sigma \alpha^2 + \sigma \alpha^2 \sqrt{1+v}}{\sigma \alpha^2 + \sqrt{1+v} + \sigma \alpha^2 \sqrt{1+v}}\right)/2], \\
\frac{\sigma \alpha^2 + \sigma \alpha^2 \sqrt{1+v}}{\sigma \alpha^2 + \sqrt{1+v} + \sigma \alpha^2 \sqrt{1+v}}, \; v \cdot (r - rc) - (1/n) \cdot Sqrt[v+1] \\
\text{Profit0}[v_\text{-}] & := \text{DemandConsumption}[1/2, 0, v] \cdot (r - rc) - (1/n) \cdot Sqrt[v+1]
\end{align*}
\]
\[ \text{FOCPIncome} ::= \text{D[ProfitIncome}[v], v] \]
\[ \text{FOCP0} ::= \text{D[Profit0}[v], v] \]
\[ \text{Reduce}[[\text{FOCPIncome} = 0], v, \text{Reals}] \]
\[ \text{Reduce}[[\text{FOCP0} = 0], v, \text{Reals}] \]
\[ \text{UtilityGraph}[e2_, k_, v_, a_, r_, sigma_, mu_, B_] := \]
\[ \text{Exp}[-\text{a} \left( \text{Exp}\left[ -a \left( \frac{1}{2 a + 4} \left( a e2 - a e2^2 + a mu - \frac{a^2 \text{sigma}^2}{4} + \frac{1}{2} a^2 k \text{sigma}^2 - a^2 k^2 \text{sigma}^2 \right) \right) + \frac{1}{4 \sqrt{1 + v}} + \frac{a^2 k \text{sigma}^2}{2 \sqrt{1 + v}} - \frac{a^2 k^2 \text{sigma}^2}{4 \sqrt{1 + v}} + \text{Log}\left[ \frac{1}{B (1 + r)} \right] \right) \right] + B \left( -\text{Exp}\left[ -a \left( e2 - e2^2 + mu - \frac{1}{2 a + 4} \left( a e2 - a e2^2 + a mu - \frac{a^2 \text{sigma}^2}{4} + \frac{1}{2} a^2 k \text{sigma}^2 - \frac{1}{4} a^2 k^2 \right) \right) + \frac{1}{4 \sqrt{1 + v}} + \frac{a^2 k \text{sigma}^2}{2 \sqrt{1 + v}} - \frac{a^2 k^2 \text{sigma}^2}{4 \sqrt{1 + v}} + \text{Log}\left[ \frac{1}{B (1 + r)} \right] \right) \right] \right) \right) * (1 + r) \right] \right] \right) * ((\text{sigma}^2 / 4) * (1 + (1 / (\text{Sqrt}[v + 1])))) \]
\[ \text{UtilityLuckl}[v_, a_, r_, sigma_, mu_, B_] := \text{UtilityGraph}[1 / 2, 1, v, a, r, sigma, mu, B] \]
\[ \text{UtilityIncome}[v_, a_, r_, sigma_, mu_, B_] := \text{UtilityGraph}[1 / 2 - \left( \frac{a \text{sigma}^2 + a \text{sigma}^2 \sqrt{1 + v}}{a \text{sigma}^2 + \sqrt{1 + v} + a \text{sigma}^2 \sqrt{1 + v}} \right) / 2, v, a, r, sigma, mu, B] \]
\[ \text{Utility01}[v_, a_, r_, sigma_, mu_, B_] := \text{UtilityGraph}[1 / 2, 0, v, a, r, sigma, mu, B] \]
UtilityLuck[a_, r_, sigma_, mu_, B_, rc_] := UtilityLuck1[0, a, r, sigma, mu, B]
UtilityIncome[a_, r_, sigma_, mu_, B_, rc_, n_] :=
UtilityIncome1[\[
\left(\frac{-8 - 4 r - 16 a \sigma^2 - 8 a r \sigma^2 + a n r \sigma^2 - a n r c \sigma^2}{4 (2 + r) (1 + a \sigma^2)^2}\right) \left(\frac{a^3 n r \sigma^4 - a^3 n r c \sigma^4}{(2 + r) (1 + a \sigma^2)^4}\right), a, r, sigma, mu, B\]
]
Utility0[a_, r_, sigma_, mu_, B_, rc_, n_] := Utility01[\[
\left(\frac{-8 - 4 r + a n r \sigma^2 - a n r c \sigma^2}{8 + 4 r}\right)\left(\frac{a^3 n r \sigma^4 - a^3 n r c \sigma^4}{(2 + r) (1 + a \sigma^2)^4}\right), a, r, sigma, mu, B\]
]
Needs["PlotLegends"]
Plot[
Table[
Utility0[i, 0.1, \(\text{sigma}^\{(1/2)\}\)], 0, 0.9, 0, i], {i, 10000, 100000, 20000}]
Table[
UtilityIncome[i, 0.1, \(\text{sigma}^\{(1/2)\}\)], 0, 0.9, 0, i], {i, 10000, 100000, 20000}]
{i, 0, 1}, PlotLabel -> Style["n: Population", 16, Bold, Black],
PlotStyle -> {Dashed, Blue},
AxesLabel -> {Style["\(\sigma^2\)", 14, Bold, Black], Style["Utility", 14, Bold, Black]},
PlotLegend -> {"D^0", "D^n"}, LegendPosition -> {-0.82, -0.55},
LegendSize -> {0.40, 0.30}, LegendShadow -> None, LegendBorder -> None,
ShadowBackground -> Opacity[0], LegendBackground -> Opacity[0]
Plot[
Table[
Utility0[i, 0.1, \(\text{sigma}^\{(1/2)\}\)], 0, 0.9, 0, i], {i, 0, 0.09, 0.01}]
Table[
UtilityIncome[i, 0.1, \(\text{sigma}^\{(1/2)\}\)], 0, 0.9, 0, i], {i, 0.09, 0.01}]
{i, 0, 1}, PlotLabel -> Style["rc: Cost of Funds", 16, Bold, Black],
PlotStyle -> {Dashed, Blue},
AxesLabel -> {Style["\(\sigma^2\)", 14, Bold, Black], Style["Utility", 14, Bold, Black]},
PlotLegend -> {"D^0", "D^n"}, LegendPosition -> {-0.82, -0.55},
LegendSize -> {0.40, 0.30}, LegendShadow -> None, LegendBorder -> None,
ShadowBackground -> Opacity[0], LegendBackground -> Opacity[0]
Plot[
Table[
Utility0[i, 0.1, \(\text{sigma}^\{(1/2)\}\)], 0, i, 0, 10000], {i, 0.1, 0.9, 0.2}]
Table[
UtilityIncome[i, 0.1, \(\text{sigma}^\{(1/2)\}\)], 0, i, 0, 10000], {i, 0.1, 0.9, 0.2}]
{i, 0, 1}, PlotLabel -> Style["\(\beta\): Time Preference", 16, Bold, Black],
PlotStyle -> {Dashed, Blue},
AxesLabel -> {Style["\(\sigma^2\)", 14, Bold, Black], Style["Utility", 14, Bold, Black]},
PlotLegend -> {"D^0", "D^n"}, LegendPosition -> {-0.82, -0.55},
LegendSize -> {0.40, 0.30}, LegendShadow -> None, LegendBorder -> None,
ShadowBackground -> Opacity[0], LegendBackground -> Opacity[0]
Plot[
Table[
Utility0[i, 0.1, \(\text{sigma}^\{(1/2)\}\)], 0, i, 0, 10000], {i, -1, 0.5}]
Table[
UtilityIncome[i, 0.1, \(\text{sigma}^\{(1/2)\}\)], 0, i, 0, 10000], {i, -1, 0.5}]
{i, 0, 1}, PlotLabel -> Style["\(\mu\): Systemic Luck", 16, Bold, Black],
PlotStyle -> {Dashed, Blue},
AxesLabel -> {Style["\(\sigma^2\)", 14, Bold, Black], Style["Utility", 14, Bold, Black]},
PlotStyle -> {Dashed, Blue}, PlotLegend -> {"D^0", "D^n"}, LegendPosition -> {-0.82, -0.55},
LegendSize -> {0.40, 0.30}, LegendShadow -> None, LegendBorder -> None,
ShadowBackground -> Opacity[0], LegendBackground -> Opacity[0]
Plot[
Table[
Utility0[i, i, \(\text{sigma}^\{(1/2)\}\)], 0, 0.9, 0, i], {i, 0.01, 0.1, 0.01}]
Table[
UtilityIncome[i, i, \(\text{sigma}^\{(1/2)\}\)], 0, 0.9, 0, i], {i, 0.01, 0.1, 0.01}]
{i, 0, 1}, PlotLabel -> Style["\(\tau\): Interest Rate", 16, Bold, Black],
PlotStyle -> {Dashed, Blue},
AxesLabel -> {Style["\(\sigma^2\)", 14, Bold, Black], Style["Utility", 14, Bold, Black]},
PlotStyle -> {Dashed, Blue}, AxesOrigin -> {0, -2.2}, PlotLegend -> {"D^0", "D^n"},
LegendPosition -> {-0.82, -0.55}, LegendSize -> {0.40, 0.30}, LegendShadow -> None,
LegendBorder -> None, ShadowBackground -> Opacity[0], LegendBackground -> Opacity[0]
Utility

$n$: Population

Utility

$rc$: Cost of Funds

Utility

$\beta$: Time Preference

Utility

$\mu$: Systemic Luck

Utility
Bank Profits

Demand1[e2_, k_, v_, a_, r_, mu_, sigma_, B_] :=
\[
\frac{1}{2 a + r} \left( \frac{a e 2 - a e 2^2 + a \mu - \frac{a^2 \sigma^2}{4} + \frac{1}{2} a^2 k \sigma^2 - \frac{1}{4} a^2 k^2 \sigma^2 - \frac{a^2 \sigma^2}{4} + \frac{a^2 k \sigma^2}{2} - \frac{a^2 k^2 \sigma^2}{4} + \text{Log} \left[ \frac{1}{B (1 + r)} \right] \right)
\]

ProfitLuckGraph1[v_, r_, rc_, n_, a_, sigma_, mu_, B_] :=
Demand1[1/2, 1, v, a, r, mu, sigma, B] * (r - rc) - (1/n) * Sqrt[v + 1]

ProfitIncomeGraph1[v_, r_, rc_, n_, a_, sigma_, mu_, B_] :=
Demand1[1/2 - \left( \left( \frac{a \sigma^2 + a \sigma^2 \sqrt{1 + v}}{a \sigma^2 + \sqrt{1 + v} + a \sigma^2 \sqrt{1 + v}} \right) / 2 \right),
\frac{a \sigma^2 + a \sigma^2 \sqrt{1 + v}}{a \sigma^2 + \sqrt{1 + v} + a \sigma^2 \sqrt{1 + v}}, v, a, r, mu, sigma, B] * (r - rc) - (1/n) * Sqrt[v + 1]

Profit0Graph1[v_, r_, rc_, n_, a_, sigma_, mu_, B_] :=
Demand1[1/2, 0, v, a, r, mu, sigma, B] * (r - rc) - (1/n) * Sqrt[v + 1]

ProfitLuckGraph[a_, r_, sigma_, mu_, B_, rc_, n_] :=
ProfitLuckGraph[0, r, rc, n, a, sigma, mu, B]

ProfitIncomeGraph[a_, r_, sigma_, mu_, B_, rc_, n_] :=
ProfitIncomeGraph[\left( \frac{-8 - 4 r - 16 a \sigma^2 - 8 a r \sigma^2 + a n r \sigma^2 - a n r \sigma^2}{4 (2 + r) \left( 1 + a \sigma^2 \right)^2} \right), r, rc, n, a, sigma, mu, B]

Profit0Graph[a_, r_, sigma_, mu_, B_, rc_, n_] :=
Profit0Graph[\left( \frac{-8 - 4 a n r \sigma^2 - a n r \sigma^2}{8 + 4 r} \right), r, rc, n, a, sigma, mu, B]
Plot[
  {Table[Profit0Graph[1, 0.1, (sigma)^(1/2), 0, 0.9, 0, i], {i, 10000, 100000, 20000}],
   Table[ProfitIncomeGraph[1, 0.1, (sigma)^(1/2), 0, 0.9, 0, i],
     {i, 10000, 10000, 20000}], {sigma, 0, 1}],
  PlotLabel -> Style["n: Population", 16, Bold, Black], PlotStyle -> {Dashed, Blue},
  AxesLabel -> {Style["σ^2", 14, Bold, Black], Style["Profit", 14, Bold, Black]},
  PlotLegend -> {"D^0", "D^1"}, LegendPosition -> {-0.82, -0.55},
  LegendSize -> (0.40, 0.30), LegendShadow -> None, LegendBorder -> None,
  ShadowBackground -> Opacity[0], LegendBackground -> Opacity[0]]
r: Interest Rate

\[
\text{Profit}
\]

\[D^p\]

\[D^r\]
Chapter 3 Without Information Production

\[ \text{Healthy}[c_2, k_] := \frac{1}{4 a \sqrt{\pi} + 2 a \sqrt{\pi} \ k} \]

\[ \text{Contract} = \frac{1}{2} a e^2 \sqrt{\pi} - 2 a e^2 \sqrt{\pi} + 2 a \mu \sqrt{\pi} - \sqrt{2} a k \sigma_2 - a^2 \sqrt{\pi} \sigma_2^2 + 2 \sqrt{\pi} \ Log \left( \frac{1}{B (1 + r)} \right) \]

\[ \text{UtilityBuyBack}[c_1, e_2, k_] := -\text{Exp}[-a + c_1] + B * \left( -\text{Exp}[-a + \left( e_2 - e_2^2 + 2 + c_1 + (1 + r) - \frac{k \sigma_2}{\sqrt{2} \pi} \right)] \right) * \text{Exp}[(\left( (1 - k) + a \right)^2 * (\sigma_2^2) / 2)]

\[ \text{FOC}[c_1] := D[\text{UtilityBuyBack}[c_1, e_2, k], c_1] \]

\[ \text{Reduce}[\{D[\text{UtilityBuyBack}[c_1, e_2, k], c_1] = 0, a > 0, B < 1, B > 0, \sigma_2 > 0\}, c_1, \text{Reals}] \]

\[ a > 0 \&\& 0 < B < 1 \&\& r > -1 \&\& \sigma_2 > 0 \]

\[ c_1 = \frac{1}{4 a \sqrt{\pi} + 2 a \sqrt{\pi} \ k} \left( 2 a e^2 \sqrt{\pi} - 2 a e^2 \sqrt{\pi} + 2 a \mu \sqrt{\pi} - \sqrt{2} a k \sigma_2 - a^2 \sqrt{\pi} \sigma_2^2 + 2 a^2 k \sqrt{\pi} \sigma_2^2 - a^2 k^2 \sqrt{\pi} \sigma_2^2 + 2 \sqrt{\pi} \ Log \left( \frac{1}{B (1 + r)} \right) \right) \]
Demand[e2_, k_] := \[\frac{1}{4 a \sqrt{\pi} + 2 a \sqrt{\pi} r} \cdot \left(2 a e2 \sqrt{\pi} - 2 a e2^2 \sqrt{\pi} + 2 a \mu \sqrt{\pi} - \sqrt{2} \ a k \sigma - a^2 \sqrt{\pi} \ \sigma^2 + 2 a^2 k \sqrt{\pi} \ \sigma^2 - a^2 k^2 \sqrt{\pi} \ \sigma^2 + 2 \sqrt{\pi} \ \log \left(\frac{1}{B (1 + r)}\right)\right) \]

BankLuck := D[Demand[1/2, k] * (r - rc), k]
BankIncome := D[Demand[1/2 - (k/2), k] * (r - rc), k]
Bank0 := D[Demand[1/2, 0] * (r - rc), k]

Reduce[{BankLuck == 0, sigma > 0, r > rc}, k, Reals]
Reduce[{BankIncome == 0, sigma > 0, r > rc}, k, Reals]
Reduce[Bank0 == 0, k, Reals]
UtilityK[a2_, k_, sigma_, mu_, a_, B_, r_] :=
- Exp[-a + 1
4 a \sqrt{\pi} + 2 a \sqrt{\pi} r (2 a e2 \sqrt{\pi} - 2 a e2^2 \sqrt{\pi} + 2 a mu \sqrt{\pi} - \sqrt{2} \ a k sigma - 
a^2 \sqrt{\pi} sigma^2 + 2 a^2 k \sqrt{\pi} sigma^2 - a^2 k^2 \sqrt{\pi} sigma^2 + 2 \sqrt{\pi} Log[1
B (1 + r)])] +
B * (-Exp[-a + 1
4 a \sqrt{\pi} + 2 a \sqrt{\pi} r (2 a e2 \sqrt{\pi} - 2 a e2^2 \sqrt{\pi} + 
2 a mu \sqrt{\pi} - \sqrt{2} \ a k sigma - a^2 \sqrt{\pi} sigma^2 + 2 a^2 k \sqrt{\pi} sigma^2 - 
a^2 k^2 \sqrt{\pi} sigma^2 + 2 \sqrt{\pi} Log[1
B (1 + r)])] * (1 + r) -

k * sigma
\sqrt{2 \pi}
)* Exp[((1 - k) * a)^2 * (sigma^2) / 2]

UtilityKLuck := D[UtilityK[1/2, k, sigma, mu, a, B, r], k]
UtilityKIncome := D[UtilityK[(1/2) - (k/2)], k, sigma, mu, a, B, r], k]
UtilityK0 := D[UtilityK[1/2, 0, sigma, mu, a, B, r], k]

Reduce[{UtilityKLuck = 0, sigma > 0, B > 0, B < 1}, k, Reals]
Reduce[{UtilityKIncome = 0, sigma > 0, a > 0, B > 0, B < 1}, k, Reals]
Reduce[{UtilityK0 = 0, k, Reals}]

\{a = 0 \&\& 0 < B < 1 \&\& r > -1 \&\& sigma > 0\} ||
\{a \neq 0 \&\& 0 < B < 1 \&\& r > -1 \&\& sigma > 0 \&\& k = \frac{-\sqrt{2} + 2 a \sqrt{\pi} sigma}{2 a \sqrt{\pi} sigma}\}

a > 0 \&\& 0 < B < 1 \&\& r > -1 \&\& sigma > 0 \&\& k = \frac{-\sqrt{2} sigma + 2 a \sqrt{\pi} sigma^2}{\sqrt{\pi} + 2 a \sqrt{\pi} sigma^2}

True

Reduce[\frac{-\sqrt{2} + 2 a \sqrt{\pi} sigma}{2 a \sqrt{\pi} sigma} = \frac{-\sqrt{2} + 2 a \sqrt{\pi} sigma}{2 a \sqrt{\pi} sigma}]

Reduce[\frac{-\sqrt{2} sigma + 2 a \sqrt{\pi} sigma^2}{\sqrt{\pi} + 2 a \sqrt{\pi} sigma^2} = \frac{-\sqrt{2} sigma + 2 a \sqrt{\pi} sigma^2}{\sqrt{\pi} + 2 a \sqrt{\pi} sigma^2}]

True

Comparison with chapter 2
\[
\text{Reduce}\left[\frac{-\sqrt{2} + 2a \sqrt{\pi} \, \sigma}{2a \sqrt{\pi} \, \sigma} < 1\right]
\]

\[
\text{Reduce}\left[\left\{\frac{-\sqrt{2} \, \sigma + 2a \sqrt{\pi} \, \sigma^2}{\sqrt{\pi} + 2a \sqrt{\pi} \, \sigma^2} < \frac{(2a * \sigma^2)}{(1 + 2a * \sigma^2)}\right\}\right]
\]

\[(\sigma < 0 \&\& \, a < 0) \lor (\sigma > 0 \&\& \, a > 0)\]

\[
\left\{\begin{array}{l}
\sigma < 0 \&\& \, a < -\frac{1}{2 \, \sigma^2}
\\
\sigma > 0 \&\& \, a > -\frac{1}{2 \, \sigma^2}
\end{array}\right\}
\]

Welfare Comparison with Chapter 2: Debtor

\[
\text{UtilityCH3Luck}[\sigma_-, \mu_-, a_-, B_-, r_] := \\
\text{UtilityK}\left[1/2, \frac{-\sqrt{2} + 2a \sqrt{\pi} \, \sigma}{2a \sqrt{\pi} \, \sigma}, \sigma, \mu, a, B, r\right]
\]

\[
\text{UtilityCH3Income}[\sigma_-, \mu_-, a_-, B_-, r_] := \text{UtilityK}\left[
1/2, \frac{-\sqrt{2} \, \sigma + 2a \sqrt{\pi} \, \sigma^2}{\sqrt{\pi} + 2a \sqrt{\pi} \, \sigma^2}, \sigma, \mu, a, B, r\right]
\]

\[
\text{UtilityCH30}[\sigma_-, \mu_-, a_-, B_-, r_] := \text{UtilityK}[1/2, 0, \sigma, \mu, a, B, r]
\]

\[
\text{DemandCH3}[e2_-, k_-, \sigma_-, \mu_-, a_-, B_-, r_] := \\
\frac{1}{4 \, a \sqrt{\pi} + 2a \sqrt{\pi} \, r} \left(2 \, a \, e2 \, \sqrt{\pi} - 2 \, a \, e2^2 \sqrt{\pi} + 2 \, a \, \mu \, \sqrt{\pi} - \sqrt{2} \, a \, k \, \sigma - a^2 \sqrt{\pi} \, \sigma^2 + 2 \, a^2 \, k \, \sqrt{\pi} \, \sigma^2 - a^2 \, k^2 \, \sqrt{\pi} \, \sigma^2 + 2 \, \sqrt{\pi} \, \log\left[\frac{1}{B(1 + r)}\right]\right)
\]

\[
\text{DemandCH3Luck}[\sigma_-, \mu_-, a_-, B_-, r_] := \\
\text{DemandCH3}\left[1/2, \frac{-\sqrt{2} + 2a \sqrt{\pi} \, \sigma}{2a \sqrt{\pi} \, \sigma}, \sigma, \mu, a, B, r\right]
\]

\[
\text{DemandCH3Income}[\sigma_-, \mu_-, a_-, B_-, r_] := \text{DemandCH3}\left[
1/2, \frac{-\sqrt{2} \, \sigma + 2a \sqrt{\pi} \, \sigma^2}{\sqrt{\pi} + 2a \sqrt{\pi} \, \sigma^2}, \sigma, \mu, a, B, r\right]
\]

\[
\text{DemandCH30}[\sigma_-, \mu_-, a_-, B_-, r_] := \text{DemandCH3}\left[1/2, 0, \sigma, \mu, a, B, r\right]
\]
UtilityCH2[e2_, k_, sigma_, mu_, a_, B_, r_] := 
-Exp[-a*(-Exp[-a*(-Exp[-a*(2*a*e2 - 2*a*e2^2 + 2*a*mu - a^2*sigma^2 + 
2*a^2*k*sigma^2 - a^2*k^2*sigma^2 + 2*Log[1/(B*(1 + r))]])]) + 
B*(-Exp[-a*(e2 + mu - (1/(4*a + 2*a))]*(2*a*e2 - 2*a*e2^2 + 2*a*mu - a^2*sigma^2 + 
a^2*k*sigma^2 - a^2*k^2*sigma^2 + 2*Log[1/(B*(1 + r))]))) * 
(1 + r) - e2^2])]*Exp[(((a*(1-k))^2)/2)*((sigma^2))]}

UtilityCH2Luck[sigma_, mu_, a_, B_, r_] := UtilityCH2[1/2, 1, sigma, mu, a, B, r]
UtilityCH2Income[sigma_, mu_, a_, B_, r_] := 
UtilityCH2[1/2 - ((2*a*sigma^2)/(1 + 2*a*sigma^2)), 
(2*a*sigma^2)/(1 + 2*a*sigma^2), sigma, mu, a, B, r]
UtilityCH20[sigma_, mu_, a_, B_, r_] := UtilityCH2[1/2, 0, sigma, mu, a, B, r]

DemandCH2[e2_, k_, sigma_, mu_, a_, B_, r_] := 
(1/(4*a + 2*a))*(2*a*e2 - 2*a*e2^2 + 2*a*mu - a^2*sigma^2 + a^2*k*sigma^2 - a^2*k^2*sigma^2 + 2*Log[1/(B*(1 + r))]])

DemandCH2Luck[sigma_, mu_, a_, B_, r_] := DemandCH2[1/2, 1, sigma, mu, a, B, r]
DemandCH2Income[sigma_, mu_, a_, B_, r_] := 
DemandCH2[1/2 - ((2*a*sigma^2)/(1 + 2*a*sigma^2)), 
(2*a*sigma^2)/(1 + 2*a*sigma^2), sigma, mu, a, B, r]
DemandCH20[sigma_, mu_, a_, B_, r_] := DemandCH2[1/2, 0, sigma, mu, a, B, r]

Needs["PlotLegends"]
\(\beta\): Time Preference

\begin{verbatim}
Plot[{Table[UtilityCH20[\(\sigma^2\)], \(\sigma^2\) \((1/2), 0, 1, 0.1, \{i, 0.1, 0.9, 0.2\}\)],
    Table[UtilityCH2Income[\(\sigma^2\)], \(\sigma^2\) \((1/2), 0, 1, 0.1, \{j, 0.1, 0.9, 0.2\}\)],
    \(\{\sigma^2, 0, 1\}\), PlotLabel \rightarrow Style["\(\beta\): Time Preference", 16, Bold, Black],
    PlotStyle \rightarrow \{Dashed, Blue\},
    AxesLabel \rightarrow \{Style["\(\sigma^2\)", 14, Bold, Black], Style["Utility", 14, Bold, Black]\},
    PlotLegend \rightarrow \{"\(D^0\)", "\(D^V\)\}, LegendPosition \rightarrow \{-0.82, -0.55\},
    LegendSize \rightarrow \{0.40, 0.30\}, LegendShadow \rightarrow None, LegendBorder \rightarrow None,
    ShadowBackground \rightarrow Opacity[0], LegendBackground \rightarrow Opacity[0]\}
Plot[{Table[UtilityCH20[\(\sigma^2\)], \(\sigma^2\) \((1/2), 0, 1, 0.9, 0.1, \{i, 0.1, 1, 0.2\}\)],
    Table[UtilityCH2Income[\(\sigma^2\)], \(\sigma^2\) \((1/2), 0, 1, 0.9, 0.1, \{j, 0.1, 1, 0.2\}\)],
    \(\{\sigma^2, 0, 1\}\), PlotLabel \rightarrow Style["\(\alpha\): Risk Aversion", 16, Bold, Black],
    PlotStyle \rightarrow \{Dashed, Blue\},
    AxesLabel \rightarrow \{Style["\(\sigma^2\)", 14, Bold, Black], Style["Utility", 14, Bold, Black]\},
    PlotStyle \rightarrow \{Dashed, Blue\}, PlotLegend \rightarrow \{"\(D^0\)", "\(D^V\)\}, LegendPosition \rightarrow \{-0.82, -0.55\},
    LegendSize \rightarrow \{0.40, 0.30\}, LegendShadow \rightarrow None, LegendBorder \rightarrow None,
    ShadowBackground \rightarrow Opacity[0], LegendBackground \rightarrow Opacity[0]\}
Plot[{Table[UtilityCH20[\(\sigma^2\)], \(\sigma^2\) \((1/2), 1, 0.9, 0.1, \{i, -1, 1, 0.5\}\)],
    Table[UtilityCH2Income[\(\sigma^2\)], \(\sigma^2\) \((1/2), 1, 0.9, 0.1, \{j, -1, 1, 0.5\}\)],
    \(\{\sigma^2, 0, 1\}\), PlotLabel \rightarrow Style["\(\mu\): Systemic Luck", 16, Bold, Black],
    PlotStyle \rightarrow \{Dashed, Blue\},
    AxesLabel \rightarrow \{Style["\(\sigma^2\)", 14, Bold, Black], Style["Utility", 14, Bold, Black]\},
    PlotStyle \rightarrow \{Dashed, Blue\}, PlotLegend \rightarrow \{"\(D^0\)", "\(D^V\)\}, LegendPosition \rightarrow \{-0.82, -0.55\},
    LegendSize \rightarrow \{0.40, 0.30\}, LegendShadow \rightarrow None, LegendBorder \rightarrow None,
    ShadowBackground \rightarrow Opacity[0], LegendBackground \rightarrow Opacity[0]\}
Plot[{Table[UtilityCH20[\(\sigma^2\)], \(\sigma^2\) \((1/2), 0, 1, 0.9, 1\)],
    Table[UtilityCH2Income[\(\sigma^2\)], \(\sigma^2\) \((1/2), 0, 1, 0.9, 1\)],
    \(\{\sigma^2, 0, 1\}\), PlotLabel \rightarrow Style["\(i\): Interest Rate", 16, Bold, Black],
    PlotStyle \rightarrow \{Dashed, Blue\},
    AxesLabel \rightarrow \{Style["\(\sigma^2\)", 14, Bold, Black], Style["Utility", 14, Bold, Black]\},
    PlotStyle \rightarrow \{Dashed, Blue\}, AxesOrigin \rightarrow \{0, -2.2\}, PlotLegend \rightarrow \{"\(D^0\)", "\(D^V\)\},
    LegendPosition \rightarrow \{-0.82, -0.55\}, LegendSize \rightarrow \{0.40, 0.30\}, LegendShadow \rightarrow None,
    LegendBorder \rightarrow None, ShadowBackground \rightarrow Opacity[0], LegendBackground \rightarrow Opacity[0]\]
\end{verbatim}

Utility

\[\begin{array}{c}
\sigma^2 \\
0.2 \\
0.4 \\
0.6 \\
0.8 \\
1.0 \\
\end{array}\]

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